JIML
JOURNAL OF INNOVATIVE MATHEMATICS LEARNING

(JIML) JOURNAL OF INNOVATIVE MATHEMATICS LEARNING
Volume 5, No. 4, December 2022

P-ISSN 2621-4733
E-ISSN 2621-4741

https://dx.doi.org/10.22460/jiml.v5i4.p165-173

STUDENTS' UNDERSTANDING OF ALGEBRAIC FACTORIZATION PROBLEMS BASED ON THEIR MEANING OF THE EQUALS SIGN

Lia Ardiansari[1], Didi Suryadi[2], Dadan Dasari[3]

[1*]Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No.229, Isola, Kota Bandung, Jawa Barat, Indonesia
liaardiansari@upi.edu

[2]Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No.229, Isola, Kota Bandung, Jawa Barat, Indonesia
ddsuryadi1@gmail.com

[3]Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No.229, Isola, Kota Bandung, Jawa Barat, Indonesia
dadan.dasari@gmail.com

ARTICLE INFO

Article history:

Received Sep 13, 2022
Revised Dec 21, 2022
Accepted Dec 24, 2022

Keywords:

Algebra
Factorization
Equal Sign

ABSTRACT

This paper reports an investigation of students' understanding of the concept of factorization in algebraic forms learned in secondary school. A total of 31 grade 8th junior high school students were selected by purposive sampling technique as respondents in this case study research. The case study approach is considered in accordance with the objectives of this study, namely obtaining in-depth knowledge of student problems, knowing the causes of these problems, and efforts that can be made to help overcome them. Methods of collecting data using written tests, interviews, and documentation. Data analysis techniques include data reduction, data presentation, and drawing conclusions. The results showed that there was a misunderstanding of students' concepts about the concept of factorization in algebraic forms so that they experienced difficulties in "algebraic manipulation". One of the causes of these difficulties is the lack of students' understanding of the equals sign as a sign of equality. The principle of mathematical equivalence serves as the main link between arithmetic and algebra. The operation transformation and the meaning of the equals sign in arithmetic as equality can underlie "algebraic manipulation".

Corresponding Author:

Lia Ardiansari,
Department of Mathematics Education,
Universitas Pendidikan Indonesia,
Jl. Dr. Setiabudhi No. 229, Bandung, Indonesia
Email: liaardiansari@upi.edu

How to Cite:

Ardiansari, L., Suryadi, D., Dasari, D. (2022). Students' Understanding of Algebraic Factorization Problems based on Their Meaning of The Equals Sign. *JIML,* 5(4), 165-173.

INTRODUCTION

Mathematics is a science that relates or examines abstract forms or structures and the relationships between these things. In order to understand structures and relationships, an understanding of the concepts contained in mathematics is required. Learning mathematics means learning about the concepts and structures contained in the subject being studied and looking for relationships between these concepts and structures. Thus, learning mathematics is basically a process of solving problems mathematically which can be used as a means to instill the habit of reasoning in one's mind. This reasoning ability makes mathematics a tool to develop critical, systematic, logical and creative thinking.

According to the curriculum used in Indonesia, algebra is one of the materials that is being introduced to SMP Class VII Semester 1 students. When students start learning algebra, they begin to enter the gateway to abstract thinking. Therefore, students begin to experience significant changes in their thinking processes, namely from arithmetic thinking to algebraic (abstract) thinking. Elly & Adam (2012) state that algebra is students' first experience with high-level mathematics. Carraher, et al. (2006) stated that algebra has long been viewed as a potential gateway for the success of other branches of higher mathematics.

Traditional school algebra is primarily a very rigid and abstract branch of mathematics and is often presented to students as a predetermined mathematical topic with strict rules, leaving no room for its own input (Ann van Amerom, 2002). Many students often see algebra as an isolated formal system in which the manipulation of symbols and rules dominates (Kieran, 2004). Algebraic factorization is one of the important algebraic materials because it is the basis for mastering algebraic equations. In the problem-solving approach to algebraic thinking, equations are a problem-solving tool, therefore constructing suitable equations is considered to be very important. Some mathematicians (such as Carraher, et al. 2006; Fuchs, 2014; McNeil, et al., 2011) agree that one of the most important concepts for developing school students' algebraic thinking is mathematical equations. The understanding of equality cannot be separated from the concept of the equals sign, so that the concept of the equals sign and the concept of equality cannot be separated (Baiduri, 2015). The equal sign is important because it is the basis for being able to understand the equation where the equation is the core of understanding algebra, therefore a correct understanding of the equal sign has important urgency for success in algebra (Banerjee, 2011).

One of the important algebraic materials to be learned by Grade VIII students of junior high school is algebraic factorization because it serves as a basis for mastering further material, one of which is Quadratic Equations such as x^2+2x-3=0. Students will have difficulty solving quadratic equation problems if students do not master algebraic factorization beforehand. However, in practice there are many research results that report learning difficulties and low learning outcomes from algebraic factorization. In solving algebraic factorization problems, it is necessary for students to be able to use operations on at least one set of mathematical objects involving the basic structure, use, and relation of operations to operations in an equation where it is considered something complicated and difficult for most students. tudents' difficulties can be seen from the mistakes they make in solving problems related to algebraic factorization. Several errors in solving algebraic factorization problems have been reported (eg Ayu, et.al., 2019; Dewanti et. al., 2018; Khatimah & Asdarina, 2020; Nugraha, et. al., 2019) include errors in understanding the concept factoring algebraic terms, simplifying algebraic fractions, and completing algebraic fractional operations, mistakes in determining strategies and solving steps on algebraic factorization, simplifying algebraic fractions, and completing algebraic fractional operations. Several national research results (such as Albaar, 2017; Minggele, 2019; Nawafilah, 2017; Nurwulan, 2021) report that the number of students who have scores below the minimum passing criteria is less than 50%. Albaar (2017) claims that the low student learning outcomes are caused by students not mastering the prerequisite material, no student collaboration in groups, lack of communication between friends, teachers are less innovative in teaching, lack of teaching materials, learning media about algebraic factoring have not been made because did not participate in teacher competency improvement activities.

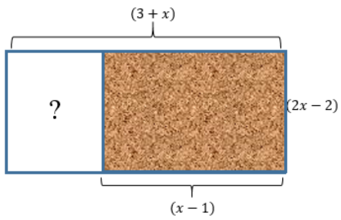
Algebraic factorization is one of the topics that has been widely used as the focus of research in mathematics education. Several research reports that raised the topic of algebraic factorization in Indonesia in the last ten years include the following. Albaar, (2017) conducted classroom action research to improve understanding of mathematical concepts on the concept of algebraic factorization using the Teams Game Tournament (TGT) learning model; Ayu, et al. (2019) describes the understanding of mathematical concepts of students who have low, medium and high learning difficulties in solving algebraic factorization problems; Dewanti, et al. (2018) describes students' understanding of mathematical concepts in solving the factorization of algebraic terms through student learning difficulties; Dewi (2014) describes the types of student errors in the algebraic factorization material; Handayani (2015) examined the effect of the giving qiestions and getting answer type of active learning model on understanding the concept of algebraic factorization; Minggele (2019) developed a STAD type cooperative learning device to increase learning motivation in the factorization of algebraic terms; Nawafilah (2017) describes the level of students' critical thinking skills in solving algebraic factorization problems; Nurwulan (2021) tested the effect of giving formative tests on mathematics learning outcomes in the factorization of algebraic terms; Putra, et al., (2017) developed a didactic design on algebraic form factoring material; Zahid & Sujadi, (2017) describe the formation of the concept of algebraic factorization of high-ability students with the APOS theory point of view.

Based on this description, it appears that the topic of algebraic factorization has been explored and researched from various aspects such as types of difficulties, causes, and various alternative solutions. However, all of the research mentioned above has not examined the fundamental concept which is the basic foundation of understanding algebraic factorization problems, namely the students' meaning of the equal sign in algebraic equations. Therefore, this study wants to take this important role, namely to explore information about the meaning of the equals sign when solving algebraic factorization problems.

METHOD

The research method used is a qualitative research method with a case study approach. In this study, the case study design used is a multi-case or multiple case study design, namely research that uses more than one case. This aims to obtain more detailed data, so that the description of the research results is more detailed and in-depth. This design is also used to generalize the resulting concept or theory. So that the use of multiple cases can cover the weaknesses found in the use of single cases which are considered not generalizable.

Table **1**. Test Questions Grid

Indicator	**Description**
Decompose the algebraic form $ax^2 + bx + c$ with $a = 1$, becoming the product of the factors.	A number, when added to three, is then squared, after which 25 is subtracted to give an algebraic form. Write down the algebraic form then factor it into the simplest form!
Decompose the algebraic form $ax^2 + bx + c$ with $a = 1$, becoming the product of the factors then determine the value.	Known $x^2 + y^2 = 72$ and $xy = -36$. Calculate the value of $(x + y)^2$ and $(x - y)^2$.
Decompose the algebraic form $ax^2 + bx + c$ with $a \neq 1$, becoming the product of the factors.	Determine the area of the unshaded region. 
Decompose the algebraic form $ax^2 + bx + c$ with $a = 1$, becoming the product of the factors.	Is it true or false the following statement? Explain your answer! $a^2b^2 + \frac{abc}{2} + \frac{c^2}{16} = \left(ab + \frac{c}{4}\right)^2$

The participants of this study were 31 grade VIII junior high school students in Banyuwangi district, East Java who were selected using a purposive sampling technique, namely students who had daily test written test scores less than the standard minimum completeness criteria that had been set. The instruments used include written tests, interviews, and documentation. The written test consists of four questions to see the students' ability to solve problems through problem solving strategies according to George Polya, namely understanding the problem, designing a plan, completing the plan, and checking again. The purpose of giving a written test is to find out students' difficulties in writing. Through these difficulties, it can be identified misunderstandings made by students to determine the extent of their understanding of the problem of algebraic factorization (see Table 1). Interviews were conducted after a written test using a semi-structured interview guide on selected subjects to dig deeper information about the difficulties experienced by students. Document analysis was carried out by analyzing the answers to students' daily tests, students' notebooks, and textbooks commonly used in the teaching and learning process in the classroom.

Data analysis techniques from Miles, Huberman, & Saldana (2013) were used as guidelines for data reduction, data presentation, and drawing conclusions. The data reduction step is carried out by grouping students with the same or similar types of difficulties based on the results of the answers given. Students with unique or interesting answers are used as interview respondents to get further information about the difficulties they are experiencing. The presentation of the data is done by displaying the results and discussion of the research obtained from the data reduction stage. The third stage is to conclude the results and related discussions in this study.

RESULTS AND DISCUSSION

Results

The performance of all respondents in completing the task is presented in Table 2 where the first column shows the number of questions, the second column is the number of students who answered the questions, the third column is the number of students who have correct answers, and the fourth column is the number of students with incorrect answers. Analysis of student misunderstandings in solving test questions focused on answers that were still not correct. Some examples of students' answers that are not quite right are described as follows.

Table 2. Student Performance in Solving Test Questions

No	Number of Students who Answered	Correct Answer	Incorrect Answer
1	21	12	9
2	31	6	25
3	29	7	22
4	25	1	24

Problem 1

In problem 1 there were nine students with wrong answers. One example of an incorrect answer is presented in Figure 1 below.

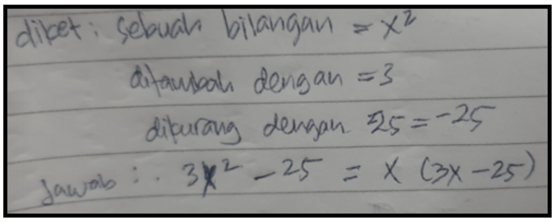


Figure 1. Example of student answers to problem 1

From these answers, it appears that students do not understand and pay close attention to the command questions. This can be seen from the students' mistakes in writing down what is known and what is asked in the question. In addition, students also have a narrow view of the meaning of the equals sign, such as writing "a number = $\boxed{x^2}$" and "$\boxed{3x^2 - 25 = x(3x - 25)}$" which indicates that the symbol puts the answer without paying attention to the equality of values. on both sides of the equals sign.

In the aspect of problem solving, there are some students who have been able to make a settlement plan by assuming the 'a number' with a variable, for example the variable x and then writing it in algebraic form. In the step of implementing the plan, which is factoring into the simplest form, there are 12 students who can complete it correctly.

Problem 2

There are several ways that can be used to solve problem number two, for example by trial and error guessing the correct value, substitution, or algebraic manipulation by applying the principles of the equal sign. However, not many students can answer this question correctly, such as one of the examples of student answers below.

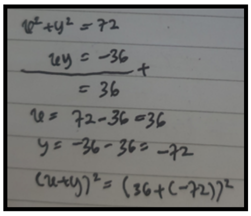


Figure 2. Example of student answers to problem 2

Based on the completion steps shown in Figure 2, it can be seen that students have not been able to understand the problem properly where the completion steps are not systematic so that the intent of each step written by students cannot be known. At the completion planning stage, to find x and y values, students add up $x^2 + y^2 = 72$ with $xy = -36$ where 36 is written as the result of the sum. It is clear that this student interprets the equals sign not as a relational sign but as a sign indicating "result".

Problem 3

Question number three has a higher level of difficulty than the two previous questions. Students must be able to understand any information given in the problem, identify it, and describe it into solving steps. In addition, students are also asked to be able to transform $(3 + x) \times (2x - 2)$ and $(x - 1) \times (2x - 2)$ into polynomial forms first. Here is one of the mistakes made by students.

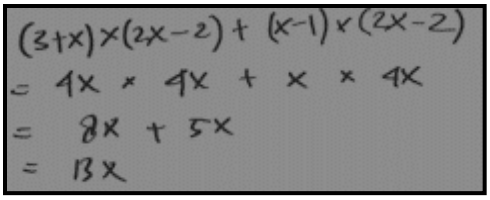


Figure 3. Example of student answers to problem 3

Based on Figure 3, you can see an example of problem solving steps carried out by students in the following three steps,

$(3 + x) \times (2x - 2) + (x - 1) \times (2x - 2)$ ……………………..step 1,
$= 4x \times 4x + x \times 4x$……………………………………………..step 2,
$= 8x + 5x$ ……………………………………………………………step 3.

From these answers, it appears that the student made a mistake in step 1 where the student wrote it as an addition, even though the area of the unshaded area is the difference. In step 2, students do not multiply $(3 + x)$ by $(2x - 2)$ and $(x - 1)$ by $(2x - 2)$, but add up each term in the algebraic form itself. In step 2, it appears that students interpret the equals sign as "producing" namely "$3 + x = 4x$" and "$x - 1 = x$". Whereas in "$2x - 2 = 4x$" students interpret the equal sign as counting "total". The error in step 2 was seen because the students did not multiply $4x$ with $4x$ and x with $4x$, but added $4x$ with $4x$ and x with $4x$. This is of course not justified because the concept of multiplying two algebraic forms is different from the concept of adding two algebraic forms. This seems to be motivated by students' views of the equals sign as a "total" count.

Problem 4

Problem 4 is slightly different from the other three questions, in that students are asked to analyze and evaluate whether the given equation is true or false. The meaning of the equals sign as a relational symbol, i.e. seeing the equals sign as a sign that states an equality relationship, is an important foundation to be used as the basis for drawing conclusions. Unfortunately, there was only 1 student who was able to answer correctly. This means that most students in this study still interpret the equal sign as an operational symbol where an equation is often read from left to right, namely the left side is a arithmetic operation and on the right side the equal sign shows the result. The following is an example of a student's answer to problem 4.

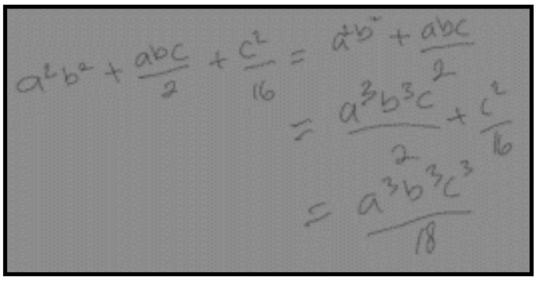


Figure 4. Example of student answers to problem 4

The student in the example answer seems to have taken the proof step from the left-hand side of the equals sign, namely $a^2b^2+\frac{abc}{2}+\frac{c^2}{16}$, then parsed it to get $\left(ab+\frac{c}{4}\right)^2$. This step seems to be based on the students' meaning of the equals sign as a sign of "producing". The student uses string operations i.e. when an equality or equation is solved using any means for example by breaking the equals sign into manageable steps to simplify calculations (such as $3+5+6=3+5=8+6=14$) . This is of course mathematically not justified because $3+5+6\neq 3+5\neq 8+6$.

Discussion

Based on the findings obtained, it can be concluded that many class 8th students in this study still do not have an adequate understanding of the problem of algebraic factorization either by describing an algebraic form (polynomial) into the form of multiplication of its factors, and vice versa. One of the underlying causes is the operational view of the equals sign which can hinder students' algebraic thinking skills. Algebraic thinking is defined by Kieran (2004) as the ability to focus on the relationships between numbers. Algebraic thinking can also be interpreted as an approach to quantitative situations that emphasizes general relational aspects with tools that are not always letter symbolization, but which can ultimately be used as cognitive support to introduce and sustain more traditional school algebra discourse.

The meaning of the equal sign in arithmetic is to announce the numerical result of a calculation, while the relational algebraic conception is to describe an equivalence state (Ann van Amerom, 2002). In the transfer from arithmetic problems to algebraic equations, the meaning of the equal sign changes from announcing a result to expressing equality. In algebra, students are asked to treat an unknown quantity as if it were a known number. In other words, operating with or on the unknown requires another idea of equivalence as well as the ability to treat unknown numbers as if they were known.

An understanding of the factorization problem can be developed by focusing on the relational meaning of the equal sign in the context of "formal equivalence" which is defined by Prediger (2010) as equality involving variables as in $\boxed{(x-2)(x+3)=x^2+x-6}$ and $\boxed{(a-b)(a+b)=a^2-b^2}$ which is satisfied for all $\boxed{x}$, $\boxed{a}$, and $\boxed{b}$. The formal equivalence of the equals sign can be interpreted in different ways depending on the interpretation of the variables involved.

CONCLUSION

The results of this study indicate that about 79% of students in this study do not yet have an adequate understanding of the problem of algebraic factorization. Many students have difficulty in developing their ideas about the notion of the equal sign as a relational symbol that expresses an equivalence relationship. Generally, students still read algebraic equations from left to right such as $\boxed{a+b=c}$ so they tend to see the equal sign in algebraic equations as a symbol for putting the answer, a sign indicating "result" or "yield", or as a "total" count. The meaning of this kind of equal sign is part of the operational view, namely seeing the equals sign as the performance of an operation.

Cultivating a relational understanding of the equals sign must begin in elementary school so that it supports algebraic thinking skills from the start. Knowledge gained from initial experience if it does not match the information found in future efforts, it will cause learning difficulties in the future. This research is still limited to analyzing students' understanding in solving algebraic factorization based on students' meaning of the equals sign, but has not explored other meanings such as letters in algebra, brackets, commutative, associative, and distributive which are also an important basis in algebraic factorization.

REFERENCES

Albaar, R. (2017). Upaya Meningkatkan Hasil Belajar Matematika Pokok Bahasan Faktorisasi Aljabar Menggunakan Model Pembelajaran Teams Game Tournamen (TGT) Siswa SMP Muhammadiyah 1 Ternate Tahun Pelajaran 2015/2016. *Humano*, 8(2),522, DOI: http://dx.doi.org/10.33387/hjp.v8i2.522.

Ann van Amerom, B. (2002). *Reinvention of early algebra : developmental research on the transition from arithmetic to algebra*. Freudenthal Instituut, Utrecht.

Ayu, N.P., Sujatmiko, P., Pramesti, G. (2019). Analisis Pemahaman Konsep Matematika Siswa dalam Menyelesaikan Soal Faktorisasi Suku Aljabar Berdasarkan Kesulitan Belajar Faktor Intelektual Siswa pada Kelas VIIIB SMPN 8 Surakarta. *Jurnal Pendidikan Matematika Dan Matematika SOLUSI*, 3(2), DOI: Https://Doi.Org/10.20961/Jpmm%20solusi.V3i2.38522.

Baiduri. (2015). Mathematics education students' understanding of equal sign and equivalent equation. *Asian Social Science*, 11(25): 15-24, https://www.researchgate.net/publication/283695347.

Banerjee, R. (2011). Is arithmetic useful for the teaching and learning of algebra?. *Contemporary Education Dialogue, 8*(2): 137–159, https://doi.org/10.1177/097318491100800202

Carraher, D., et al. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, 37(2): 87-115, L: http://www.jstor.org/stable/30034843.

Dewanti, N., Sujatmiko, P., & Pramesti, G. (2018). Analisis Pemahaman Konsep Matematika Siswa dalam Menyelesaikan Soal Faktorisasi Suku Aljabar Berdasarkan Kesulitan Belajar Faktor Intelektual Siswa Pada Kelas VIII B SMPN 8 Surakarta Tahun Ajaran 2016/2017. *Phenomenon: Jurnal Pendidikan MIPA,* 8(1), 26-35, doi:https://doi.org/10.21580/phen.2018.8.1.2492.

Dewi, S.I.K. (2014). Analisis Kesalahan Siswa Kelas Viii Dalam Menyelesaikan Soal Pada Materi Faktorisasi Bentuk Aljabar Smp Negeri 1 Kamal Semester Gasal Tahun Ajaran 2013/2014. *MATHEdunesa*, 3(2): 95-202, https://doi.org/10.26740/mathedunesa.v3n2.p%25p.

Ely, R & Adams, A.E. (2012). Unknown, placeholder, or variable: what is $\boxed{x}$?. Mathematics *Education Research Group of Australasia*, 24, 19–38, DOI:10.1007/s13394-011-0029-9.

Fuchs, L.S. et al. (2014). Does calculation or word-problem instruction provide a stronger route to pre-algebraic knowledge?. *Journal of Educational Psychology*, 106(4), 990-1006. http://dx.doi.org/10.1037/a0036793.supp

Handayani, S. (2015). Pengaruh Model Pembelajaran Aktif Tipe *Giving Questions and Getting Answer* terhadap Pemahaman Konsep Matematis Siswa Kelas VIII SMPN 10 Padang Pada Materi Pokok Operasi Aljabar dan Faktorisasi Aljabar. *Jurnal Pelangi*, 8(1):390-396. Doi: Https://Doi.Org/10.22202/Jp.2015.V8i1.390

Khatimah, H., & Asdarina, O. (2020). Diagnosis kesalahan siswa dalam memahami materi faktorisasi bentuk aljabar pada siswa kelas VIII. *Mathema Journal, 2*(1): 40-56.

Kieran, C. (2004). Algebraic Thinking in Early Grades: What Is It?. *The Mathematics Educator*, 8 (1), 139 – 151.

McNeil, N.M., et al. (2011). Benefits of practicing 4 = 2 + 2: Nontraditional problem formats facilitate children's understanding of mathematical equivalence. *Child Development, 82*(5), 1620–1633. DOI: 10.1111/j.1467-8624.2011.01622.x

Miles, M., Huberman, M., & Saldana, J. (2013). *Qualitative data analysis: a methods sourcebook: an expanded sourcebook*. 408.

Minggele, D. (2019). Pengembangan Perangkat Pembelajaran Kooperatif Tipe STAD untuk Meningkatan Motivasi Belajar Siswa Kelas VIII Pada Materi Faktorisasi Suku Aljabar. *Ekspose: Jurnal Penelitian Hukum dan Pendidikan*, 18(1): 369-379, DOI: 10.30863/ekspose.v18i1.369.

Nawafilah, N.Q. (2017). Identifikasi Kemampuan Berpikir Kritis Siswa Dalam Menyelesaikan Soal/Masalah Matematika Pada Materi Faktorisasi Suku Aljabar di Kelas VIII SMP. *Teknika*, 9(2): 189-197, DOI: https://doi.org/10.30736/teknika.v9i2.189.

Nugraha, N., Kadarisma, G., & Setiawan, W. (2019). Analisis kesulitan belajar matematika materi bentuk aljabar pada siswa SMP kelas VII. *Journal on Education, 1*(2): 323-334.

Nurwulan, S. (2021) Pengaruh Pemberian Tes Formatif Terhadap Hasil Belajar Matematika Materi Faktorisasi Suku Aljabar pada Kelas VIII Mts Qur'aniyah Batu Kuta-Narmada Lombok Barat. *Jurnal Syntax Admiration* 2(6): 1167-1175, https://doi.org/10.46799/jsa.v2i6.253.

Prediger, S. (2010). How to develop mathematics-for-teaching and for understanding: the case of meanings of the equal sign. *Journal of Mathematics Teacher Education, 13*(1), 73–93. DOI 10.1007/s10857-009-9119-y.

Putra, R. W. Y., Nurwani, N., Putra, F. G., & Putra, N. W. (2017). Pengembangan Desain Didaktis Bahan Ajar Materi Pemfaktoran Bentuk Aljabar pada Pembelajaran Matematika SMP. *Numerical: Jurnal Matematika Dan Pendidikan Matematika*, 1(2): 97–102, https://doi.org/10.25217/numerical.v1i2.133.

Zahid, M.Z. & Sujadi, I. (2017). Pembentukan Konsep Faktorisasi Aljabar Siswa Berkemampuan Tinggi. *Kreano Jurnal Matematika Kreatif-Inovatif*, 8(1): 94-100, DOI:10.15294/kreano.v8i1.7517.