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PROBLEM-SOLVING-ORIENTED TRAINING TEACHING METHOD TO IMPROVE MATHEMATICAL LOGICAL REASONING ABILITY ON MIDDLE SCHOOL STUDENTS

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ABSTRACT

With the deepening of the reform and development of the new curriculum education in the 21st century, the mathematics core accomplishment has been paid more and more attention by middle school teachers, among which, the logical reasoning ability is one of the important contents of the mathematics core accomplishment in middle school. The research purpose of this paper is to solve the problem of middle school students mathematical logical reasoning ability is not strong, starting from the mathematical problem itself, to explore how to cultivate students' logical reasoning thinking ability and teaching methods. This paper adopts the literature analysis method to summarize and analyze the relevant literature, uses the case analysis method to deeply analyze the relevant classic examples, and uses the statistical analysis method to analyze the students' logical reasoning literacy. Through the analysis of cases and students, this study believes that having the ability of mathematical logic reasoning can help students to solve mathematical problems, and similarly, it also promotes the development of students' mathematical logic reasoning ability in the process of solving problems. When cultivating students logical reasoning ability in problem-solving problems, we can start from the following aspects, 1. Pay attention to the study of basic knowledge, 2. Cultivate students' observation ability, 3. Analysis and prove with the help of innovative problems.

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INTRODUCTION

With the comprehensive deepening of the reform, people pay more attention to the overall development of students' quality as well as their achievements. After the 21st century, mathematical education circles have a more thorough understanding of logical reasoning, therefore, reasoning ability began to become one of the five basic abilities of mathematics is written into the curriculum standards(Bass 2020; WU 1999). And with the deepening of the curriculum reform, the basic ideas of mathematics gradually get the attention of the public and began to conduct in-depth research, and logical reasoning is regarded as one of the most important mathematical ideas has been widely noticed(Atit, Uttal, and Stieff 2020; Ku and Ho 2010). Under the background of the new curriculum reform, logical reasoning has become one of the six core qualities of mathematics, and runs through all middle school mathematics knowledge.The middle school stage is the key for students to rapidly improve their logical reasoning, and the logical reasoning ability of students at this stage is mainly reflected when they are faced with complex proof reasoning problems, but this kind of question is exactly the deficiency of many students, mainly examines the thinking and reasoning ability of students in the face of complex problems(Adeyemi 2012; Kay et al. 2012; Tanudjaya and Doorman 2020). So in this paper, through the logical reasoning in different forms of the case and the Angle of learning method to explore.

In the middle school mathematics teaching activities, learning mathematics can make students' knowledge and theoretical system to obtain a more comprehensive development, and for promoting students' theoretical knowledge, professional skills and other aspects can be properly integrated into the core literacy in the middle school mathematics teaching activities as the training goal(Cobb and Jackson 2011; Maass, Swan, and Aldorf 2017; Reynolds 2016; Reynolds and Harel 2011). In the process of teaching of middle school mathematics knowledge, teachers should be the logical thinking ability as a student in dealing with the key to cultivate mathematics subject, is conducive to better exploit their potential of mathematical thinking, and in the process of solving math problems for middle school students can feel the rigor of mathematical disciplines, so as to better improve their ability of logical reasoning(Carly 2010; Pea 2007; Putnam et al. 2013). Under the cultivation of logical reasoning literacy, all aspects of mathematical problems should be considered, so when students face complex problems, they can better cultivate their divergent thinking and fully mobilize their participation and enthusiasm in learning. he improvement of logical reasoning ability also benefits from the integration of knowledge from books with real life, which better trains students to see the world through mathematical eyes and their practical skills, and strengthens their grasp of the basics(Manuel et al. 2019; Rohendi 2012; Yuliani and Saragih 2015).

The Mathematics Curriculum Standards for Compulsory Education (2011 Edition) clearly states that "the development of reasoning should be integrated with the whole process of learning mathematics(Ayalon 2019). Accordingly, the General High School Mathematics Curriculum Standards (Experimental) also clearly states that the development and improvement of students' ability of deductive reasoning or logical proof is an important goal of the high school mathematics curriculum, and that the connection between syllogistic reasoning and deductive reasoning is close and complementary(Ayal and Kusuma 2016). As can be seen from the mathematics curriculum standards of countries around the world, logical reasoning is internationally recognized as a fundamental and important ability. Moreover, logical reasoning is one of the enduring research hotspots in mathematics education both at home and abroad(Jiang et al. 2021). However, there are still many problems in the research of mathematical logical reasoning. The teaching of mathematical logical reasoning is the hot spot of research, followed by the research on the other, connotation, structure and status of mathematical logical reasoning, while there is not much research on the teaching materials and curriculum of logical reasoning. Although the research on mathematical logical reasoning courses is mature on the whole, it is dominated by geometry courses, and there is very little research on logical reasoning in algebra and probability courses(Moursund n.d.; Tibshirani and Friedman 2005; Weintrop et al. 2016). Although geometry plays an irreplaceable role in the development of secondary school students' logical reasoning ability, all branches of mathematics are full of reasoning, and teaching geometry is a way to develop students' logical reasoning ability, but it is by no means the only material and way. Future research should focus on the balanced development of logical reasoning in "Mathematics and Algebra" and "Statistics and Probability". The research results in this area are still insufficient, and with the advancement of the new curriculum, the need for research in this area is becoming more and more important.

Therefore, the purpose of this paper is: 1. to address the fact that most of the articles on developing students' logical reasoning skills in mathematics focus on theoretical studies, and few of them analyze the theory in the context of our most common mathematical topics and textbooks; 2. to focus on classical examples from "Number and Algebra", "Statistics and Probability", and 3. In this paper, we focus on classic examples from "Mathematics and Algebra" and "Statistics and Probability".

Research Questions: 1. How to highlight mathematical logic ideas in classical examples? 2. How to develop students' logical reasoning skills?

THEORITICAL

Logical Reasoning

Logical reasoning is divided into: sympathetic reasoning and deductive reasoning. Sympathetic reasoning is further divided into inductive reasoning and analogical reasoning. Sympathetic reasoning leads to conjecture. Sympathetic reasoning is a cognitive process in which people use observation, experiment, induction, analogy, association, intuition, and other non-deductive ways of thinking to construct a reasonable understanding of an object based on their existing knowledge and experience and under the influence of emotion. Deductive reasoning: The process of reasoning from the general to the particular, the premise must contain the conclusion, so the conclusion must be true, that is, deductive reasoning is necessary reasoning. In "How to Solve Problems", Polya suggests that "induction is the process of discovering universal laws by observing and combining particular examples (Hensberry and Jacobbe 2012; Lee 2017; Stylianou 2002)."

Sympathetic and deductive reasoning complement each other, depend on each other, interact with each other, and together drive discovery activities most of the time, and these two important forms of thinking and reasoning are indispensable in the process of mathematical exploration and the construction of mathematical systems. Sympathetic reasoning sets the goal and direction for deductive reasoning, while deductive 3 reasoning infers and justifies the conjectures of sympathetic reasoning.

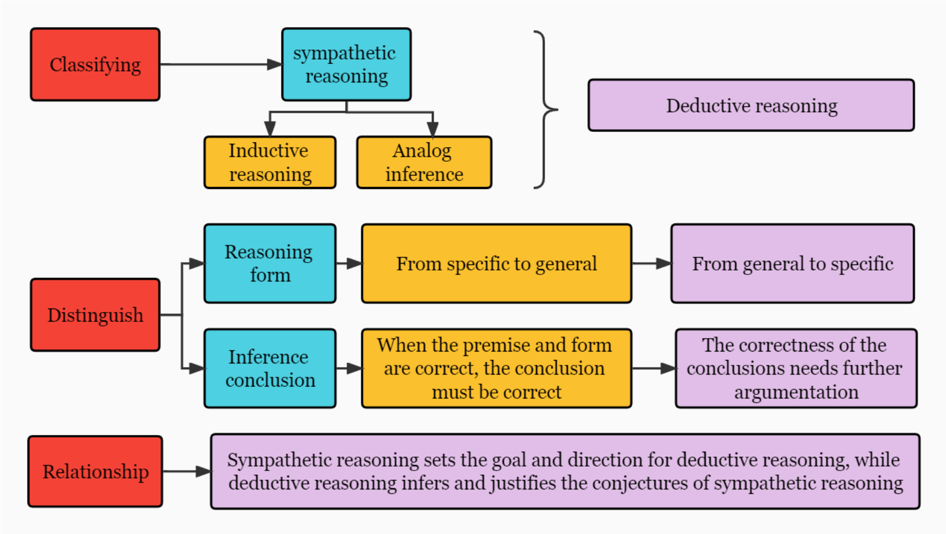

Figure 1. Relationship between sympathetic and deductive reasoning(Holyoak 1986; Matson 2017)

Mathematical Logic Reasoning

What is mathematical logical reasoning? Mathematical logical reasoning is an important part of mathematical reasoning, which refers to the students' ability to synthesize and analyze relevant mathematical problems and to reason and prove them through the form and manner of thinking that they have developed through long-term thinking patterns(M and A 2015; Missouri 1981). In other words, students learn the basics solidly and have a certain reserve of mathematical knowledge, then reserve the knowledge systematically into their knowledge network, so as to form certain logical reasoning ability, and then in the process of solving problems, form mathematical thinking, and then improve their logical reasoning ability.

METHOD

Literature analysis method

Searched keyword for this study is "logical reasoning literacy", "mathematics senior high school" and "research methods" on China National Knowledge Infrastructure (CNKI) or "We then categorized and summarized the searched data, sorted and compared the literature on mathematical logical reasoning literacy at home and abroad, and analyzed and reflected on the index system of logical reasoning literacy(Reiss 2020; Yigit 2014).

Case study method

The study analyzes classic examples of basic "number and algebra" and "geometry", and through continuous discussion with front-line teachers, analyzes and reflects on mathematical logical reasoning literacy in high school mathematics examples. This course will provide references and suggestions for the teaching of examples under the goal of developing mathematical logical reasoning literacy

Statistical analysis method

The object of this research is 11th grade senior high school students in Beihai City, Guangxi. A total of research sample are 200 students to be tested were selected. According to the recovery of test papers, 160 samples of students to be tested were finally selected for this study. Among them, there are 100 male and 60 female. In the research method, the main method is to compile the test paper and investigate, which is a combination of quantitative and qualitative. Quantitative analysis is mainly SPSS, AMOS software for statistical analysis of the collected data.

This combined quantitative and qualitative research method can not only do statistical analysis on the level of logical reasoning literacy of high school students; it can also further analyze how to improve the level of logical reasoning literacy of high school students, thus providing good suggestions for cultivating logical reasoning literacy of high school students.

RESULTS AND DISCUSSION

All formulas and theorems in secondary mathematics are derived from rigorous reasoning, and it can even be said that without mathematical reasoning there would be no mathematics teaching. In this paper, we will look at the theory of logical reasoning embodied in the classic cases, the implementation methods, and the contribution to the improvement of students' logical reasoning ability.

3.1 Application of Inductive Reasoning in Secondary School Mathematics Problem Solving

The mathematical genius Gauss once said, "Mathematics is a gymnastic exercise in thinking". And inductive reasoning and analogical reasoning play an important role in developing students' creative thinking, and are good exploratory and anticipatory for their problem solving. The proper introduction of inductive methods in the process of problem solving is not only conducive to discovering the nature of the problem and finding the laws, but also plays a very important role in improving students' understanding of the problem, their ability to analyze and solve problems, and the development of their creative thinking skills.

[Example 1] Look carefully at the pattern of number arrangement in the table below.

The second number in line 12 is______;

The second number in line $n\ (n \geq 2)$ is______.

					1						.... 1st row
				2		2					.... 2nd row
			3		4		3				.... 3rd row
		4		7		7		4			.... 4th row
	5		11		14		11		5		.... 5th row
6		16		25		25		16		5	.... 6th row
					... ...						.... n row

Figure 3. Logical reasoning framework

The above table shows that it is a deformed version of Yang Hui's triangle, but it is still essentially an application of knowledge of series.

Looking at the icons, we can see that the first row has a 1, the second row has two 2s, and the first number in the third row is 3.

If the second number of the b_{n-1} row is $n-1$, The second number in the b_n row is n. Then the law of Yang Hui's triangle gives $b_n = (n-1) + b_{n-1}$, namely

$b_n - b_{n-1} = n-1$. from $b_3 = 4$,

$$b_n = (b_n - b_{n-1}) + (b_{n-1} - b_{n-2}) + \cdots + (b_4 - b_3) + b_3 = (n-1) + (n-2) + \cdots + 4 + 3 + 4$$
$$= \frac{3+(n-3)}{2} \times (n-3) + 4 = \frac{n^2 - n + 2}{2}$$

So the second number in line 12 is $b_{12} = 67$, So the second number in line n is $b_n = \frac{n^2 - n + 2}{2}$.

[example 2] Let the function $f(x) = \frac{e^x}{x^2 + ax + a}$, Where a is a real number.

(1) Assume that $f(x)$ is defined when its domain is R, Then try to find the range of values of the real number a;

(2) Assume that $f(x)$ is defined when its domain is R, Try to find what is the monotonic decreasing interval of $f(x)$?

Solution: (1) From the title, we know that the domain of definition of $f(x)$ is R,then $x^2 + ax + a \neq 0$ constant is true,

$\therefore \Delta = a^2 - 4a < 0$, $0 < a < 4$,

So it can be found that when $0 < a < 4$, $f(x)$ has the domain of definition R .

（2）$\because f'(x) = \frac{x(x+a-2)e^x}{(x^2+ax+a)^2}$,by $f'(x) = 0$, $x = 0$ or $x = 2-a$.

$\because 0 < a < 4$,∴when $0 < a < 2$, $2-a > 0$.

∴when $(-\infty, 0)$, $f'(x) > 0$, when $(0, 2-a)$, $f'(x) < 0$, when $(2-a, +\infty)$, $f'(x) > 0$,

So the monotonicity reduction interval of $f(x)$ is $(0, 2-a)$.

When $a = 2$时, $f'(x) \geq 0$ is always true;

When $2 < a < 4$时, $2-a < 0$.

∴When $(-\infty, 2-a)$, $f'(x) > 0$, When $(2-a, 0)$, $f'(x) < 0$, When $(0, +\infty)$, $f'(x) > 0$,

So the monotonic reduction interval of $f(x)$ is $(2-a, 0)$.

In conclusion, when $0 < a < 2$, the monotonically decreasing interval of

$f(x)$ is $(0,2-a)$.

Therefore ,When $2<a<4$, the monotonic reduction interval of $f(x)$ is $(2-a,0)$.

Analysis: the first problem can be solved by students through complete induction and listing one by one. However, it is obviously very complicated to extend to line n, so you can consider using special methods to solve this problem. The difficulty of this problem increases gradually in the process of expansion, which stimulates students' thirst for knowledge, and then allows them to experience the special to general reasoning thought and reflect the thinking process of induction through observation, generalization and conclusion.

(1) Logical reasoning theory contained in the solution: the above examples reflect the reasonable reasoning in logical reasoning.

(2) Implementation method of implied logical reasoning: when solving a series of problems, students can first analyze their characteristics through observation, summarize and put forward conjectures, and finally get the correct conclusion. In the process of problem solving, students will make full use of reasonable reasoning to achieve the smooth development of their logical reasoning ability, but using reasonable reasoning alone is not enough to prove the correctness of the problem.

(3) Contribution: when providing exercises for students to solve, teachers need to simultaneously emphasize the key points of logical reasoning and students' reasonable and slightly subjective judgments. Only under this idea, students can intuitively feel the help of logical reasoning ability in building mathematical knowledge and solving mathematical problems.

3.2 application of deductive reasoning in middle school mathematics problem solving

Deductive reasoning is the necessary form of reasoning to get the correct conclusion. In the solution of mathematical problems in middle school, it is inseparable from the application of deductive reasoning, and it also widely exists in mathematical knowledge such as geometry, algebra and sequence. Deductive reasoning not only provides us with rigorous ideas for solving problems, but also the conclusions are more authentic.

[example 2] Let the function $f(x)=\dfrac{e^x}{x^2+ax+a}$, Where a is a real number.

(1) Assume that $f(x)$ is defined when its domain is R, Then try to find the range of values of the real number a ;

(2) Assume that $f(x)$ is defined when its domain is R, Try to find what is the monotonic decreasing interval of $f(x)$?

Solution:

(1) From the title, we know that the domain of definition of $f(x)$ is R,then $x^2+ax+a\neq 0$ constant is true，

$\therefore \Delta=a^2-4a<0$ ， $0<a<4$, So it can be found that when $0 < a < 4$ ， $f(x)$ has the domain of definition R .

(2) $\because f'(x) = \dfrac{x(x+a-2)e^x}{(x^2+ax+a)^2}$ ，by $f'(x)=0$ ， $x=0$ or $x=2-a$. $\because 0<a<4$,$\therefore$when $0<a<2$ ， $2-a>0$.

$\therefore$when $(-\infty,0)$ ， $f'(x)>0$ ， when $(0,2-a)$ ， $f'(x)<0$ ， when $(2-a,+\infty)$ ， $f'(x)>0$，

So the monotonic reduction interval of $f(x)$ is $(0,2-a)$.

When $a=2$时， $f'(x)\geq 0$ is always true；

When $2<a<4$时， $2-a<0$.

$\therefore$When $(-\infty,2-a)$ ， $f'(x)>0$ ， When $(2-a,0)$ ， $f'(x)<0$ ， When $(0,+\infty)$ ， $f'(x)>0$,

So the monotonic reduction interval of $f(x)$ is $(2-a,0)$.

In conclusion, when $0<a<2$ ， the monotonically decreasing interval of $f(x)$ is $(0,2-a)$.

Therefore ,When $2<a<4$ ， the monotonic reduction interval of $f(x)$ is $(2-a,0)$.

Analysis: The calculations in this question, like many other methods of reasoning, are the most basic and reliable tools we can use in solving logical reasoning problems, especially when using algebraic methods to solve some problems, which often expose the essence of the problem and lead us to adequate and reliable conclusions.

(1) The theory of logical reasoning embedded in the solution: The above example reflects the deductive reasoning in logical reasoning.

(2) Implicit logical reasoning implementation method: In general, most of the reasoning problems in algebra reflect deductive reasoning, but very often the trinomial form is simplified, and the major premises are generally omitted in the reasoning process. In deductive reasoning, trigonometry is an important form, through a certain amount of this type of exercises, students can better understand and master the concept of "trigonometry", and learn to use "trigonometry" to solve problems, which has a very important role in their deductive reasoning ability. This is a very important role for their deductive reasoning ability.

(3) Contribution: Deductive reasoning not only keeps students' thinking rigorous, but it has an irreplaceable effect on developing students' thinking consistently and logically. Experiencing a series of orderly proofs allows

students to feel the rigor of the mathematical discipline and forms their attitude of respecting science and seeking truth from facts.

3.4 Integrated Application of Sympathetic and Deductive Reasoning

Sympathetic and deductive reasoning exist together in most questions about logical reasoning. The essence of sympathetic reasoning is "discovery", while deductive reasoning can be used to verify the conclusions of sympathetic reasoning, and at the same time, sympathetic reasoning provides some direction and ideas for deductive reasoning.

[Example 3] As shown in Figure 4, the center of the ellipse can be seen to lie at the coordinate origin O，F is the left focus of the ellipse，Then when $FB \perp AB$, the eccentricity of the ellipse is $\frac{\sqrt{5}-1}{2}$, and an ellipse like this is called a "golden ellipse"。Then, can we find the eccentricity e of the "golden hyperbola" by analogy with the "golden ellipse"?

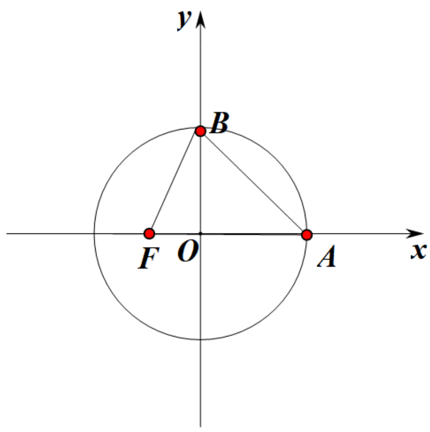

Figure 4

By analogy with the "golden ellipse", we can set the equation of the hyperbola as $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(a>0,b>0)$, then $F(-c,0), B(0,b), A(a,0)$,

So $\overrightarrow{FB}=(c,b), \overrightarrow{AB}=(-a,b)$.

It is easy to know that $\overrightarrow{FB} \perp \overrightarrow{AB}$, so $\overrightarrow{FB}\cdot\overrightarrow{AB}=b^2-ac=0$, $c^2-a^2-ac=0$, that is $e^2-e-1=0$,

And $e>1$,so $e=\frac{\sqrt{5}-1}{2}$.

Analysis: The problem is based on the known condition of "golden ellipse" to find the eccentricity of "golden hyperbola", which can improve the students' associative thinking and innovative thinking in problem solving. By making bold conjectures, students can develop their thinking and then prove their conjectures by themselves, which greatly improves their ability of logical thinking, application transfer and critical insight. The above case exemplifies:

(1) The theory of logical reasoning embedded in the solution: The above example reflects the joint application of sympathetic and deductive reasoning.

(2) Implicit logical reasoning implementation method: first make a reasonable conjecture about the topic to provide a general idea for solving the problem, and then use the proof method to prove the correctness of the conjecture.

(3) Contribution: Students' logical reasoning ability is inseparable from problem solving, and while solving mathematical problems improves students' logical reasoning ability, innovation ability, etc., making them good at thinking and taking the initiative to participate in mathematical activities. If students are able to fully understand and explain a mathematical problem, they will be able to exercise their mathematical thinking, so that when they encounter the same essential theoretical problem, they will have a fuller and faster understanding, which will effectively help them to solve the problem. In this way, not only will students' logic become more rigorous, but also their ability to analyze problems will be improved. In this way, when students are faced with difficult and complex problems, they will be able to solve these new problems through their own effective analysis.

CONCLUSION

Pay attention to the basics

The most important thing in solving mathematical problems is the amount of knowledge students already have, and this foundation helps students a lot in observing, analyzing, understanding and thinking about the topic. Lack of basic knowledge in solving problems will feel at a loss, and do not know why, not to mention the development of other thinking skills of students. Having a good foundation in mathematics will not only help students better understand and analyze topics and grasp problem solving strategies, but will also effectively improve problem solving efficiency.

Develop students' observation skills

Students' observation ability plays a very important role in developing their sympathetic reasoning. From the above 10 many cases, we can find that the problems can be observed first by using sympathetic reasoning, making conjectures and hypotheses about the problems, and then using deductive reasoning or proof to get the answers. Therefore, in the process of teaching mathematics, we should pay attention not only to the thoroughness of thinking and the accuracy of the results, but also to the intuitive exploration and discovery of thinking, which means that the rationality and necessity of mathematical sympathetic reasoning should be emphasized.

Analyze and prove with the help of innovative problems

Innovative problems, also known as open-ended problems, are one of the most educationally valuable types of problems in mathematics teaching. This is because innovative problems cover not only the richness of logical reasoning, but also emphasize the cultivation of creative thinking in students. In such problems, students usually need to go through a series of processes such as insight, understanding, analysis, verification, and induction. Therefore, when developing students' logical reasoning literacy, teachers should also pay attention to the application of open-ended problems so that students can improve their logical reasoning skills in the process of solving innovative problems.

The idea of logical reasoning in mathematics is not only embedded in mathematical knowledge, but also in various types of mathematical problems, and, logical reasoning is also an important way of thinking in mathematics. Using the topic as a guide, it is a better way to develop students' logical reasoning thinking by allowing them to focus all their attention on a problem practice, carefully observe and reason out the important conclusions in it. Focusing on cultivating students' mathematical ability in the process of problem solving lays a good foundation for subsequent learning of higher-level mathematical knowledge in order to improve students' core literacy in secondary school mathematics. Therefore, focusing on the exercise of students' logical thinking can not only improve their mathematical thinking, but also enhance their overall learning efficiency.

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