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MATHEMATICAL CREATIVE THINKING AND HABITS OF MIND GROUNDED ON STUDENT'S COGNITIVE STAGE

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Abstract

The purpose of this survey is to examine the role of cognitive stage toward his or her attaiment of mathematical creative thinking and habits of mind grounded on student's cognitive stage. The survey concerned with 36 eleventh grade students of 16.25 years old, test of logical thinking , an essay MCT test, and a MSC scale. In line with that findings, formal students realized few difficulties, transition students experienced some more difficulties and concrete students realized difficulties in all items of MCT tasks.

Keywords: cognitive stage, TOLT, mathematical creative thinking, habits of mind,.

Abstrak

Tujuan dari survei ini adalah untuk menguji peran tahap kognitif terhadap sikap berpikir kreatif matematis dan kebiasaan pikiran yang didasarkan pada tahap kognitif siswa. Survei tersebut melibatkan 36 siswa kelas sebelas berusia 16,25 tahun, tes pemikiran logis, tes MCT esai, dan skala MSC. Sejalan dengan temuan itu, siswa formal menyadari beberapa kesulitan, siswa transisi mengalami lebih banyak kesulitan dan siswa nyata menyadari kesulitan dalam semua item tugas MCT.

Kata kunci: tahap kognitif, TOLT, berpikir kreatif matematik, kebiasaan berpikir matematik.

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INTRODUCTION

To analyze student's cognitive stage, our discourse connected to a famous theory that is Piaget's theory of child cognitive development. By using in depth and persistent observations and interviews toward groups of children of various age from some excelent schools in Geneva, Inhelder and Piaget (Sumarmo 1987b)identified some cognitive abilities able and not able to do by each group of children. Those cognitive abilities involve logical reasoning abilities namely controlling variable reasoning, proportional reasoning, probabilistic reasoning, combinatorial reasoning, and propositional reasoning. Based on the ways of reasoning each group of children, then Inhelder and Piaget (Sumarmo 1987b) classified the cognitive child development into four main operational stages consecutively, namely: a. Sensory-motor operational stage (infant up to 2 years old); b. Pre- concrete operational stage (2 – 7 years old); c. Concrete operational stage (7-12 years old); d. formal operational stage (13-14 years old or 14-15 years old).

Some experts realized that implementation of Inhelder and Piaget (Sumarmo 1987b)experiments accompanied with precise observations and interviews need very long time. Therefore, Capie and Tobin (Sumarmo 1987a)developed a written test that is known as Test of logical thinking (TOLT), for determining student's cognitive development stage. The TOLT consists of ten items and covers of proportional reasoning, controlling variable reasoning, probabilistic reasoning, and combinatorial

reasoning. Further, Capie and Tobin (Sumarmo 1987b)conducted cross culture studies toward TOLT which various groups of students non Inhelder and Piaget subject's culture from sixth grade student up to college students. The study found that students of more than 15 years old had not achieved at formal stage yet, but in higher school grade, study found increased percentage of formal stage students and decreased percentage of concrete students.

Concerning on mathematics learning out comes, our discussion will relate to mathematical creative thinking (MCT). Teachers realize that MCT is one of essential mathematics learning outcomes should be comprehended by high school students. The reason of that statement not only caused of it was attached in the goal of teaching mathematics but also it related with daily life demand. The goal teaching mathematics, among other are: to improve student's potency to become a critical, creative, logical, accurate, and innovative human . Beside that, creative thinking was a part of life-skill which it is needed for overcoming daily live problems.

Some experts (Munandar n.d.)clarified creative thinking term differently, however they include four main similar components namely: fluency, flexibility, originality, and elaboration. To think over those mathematical processes involved in solving MCT task, it described that MCT was classified as higher order thinking (HOT) skill in mathematics and included difficult and complex mathematical processess. Implication of that statement among other was for completing MCT, beside student should master the mathematics contents and processes, student should have strong disposition as well, such as to be tolerant to uncertainty, having open minded attitude, having self confidence, self controll, and unafraid to take a risk, so that student was motivated to work hard and to manage his learning habit (Costa 2001b). Then, Costa(Costa 2001b)denotes that strong disposition as habits of mind (HoM). Basically, HoM in mathematics was an essential attitude in doing complex mathematics task, caused of HoM will help student to be proficient, creative, and self-relient person and having responsible toward his activities.

Some studies Damayanti, Sumarmo, and Maya (2018) by implementing various teaching approaches invented that students obtained MCT at medium to fairly good grade level. While other studies (Fitri 2017) reported different finding that was students attained HoM at fairly good grade level. Those findings pointed out that students realized few difficulties on completing MCT tasks but students did not experienced to behave HoM attitudes.

Those afformentioned arguments and findings, motivate researcher to excecute a study for analyzing student's MCT and HoM related to student's cognitive stage, and posing research questions as follow.

a. What are distribution of cognitive stage of eleventh grade students measured by implementing TOLT?
b. What are student's grade on MCT and on HoM for entirely students and according to its cognitive stage?
c. What kind of student's difficulties on solving MCT tasks?
d. Is there any association between MCT and HoM?

Theoretical Review

Stage of Child's Cognitive Development

Based on the way of children's thinking, Inhelder and Piaget (Sumarmo, 1987b) carry out a series of experiments accompanied with accurate observations and individual interviews toward a number of subjects of various age groups from children up to addolescence from various best of schools in Geneva. Based on those observations and interviews then Inhelder and Piaget (Sumarmo 1987b)identify some cognitive abilities that able to do or not able to do by each age group of children. By analyzing the structure of each age group of children, then Inhelder and Piaget (Sumarmo, 1987b) classified the subjects into four main stages of thinking or stage of child cognitive development, namely: a) Sensory-motor stage (infant up to 2 years old); b) Pre- concrete operational stage (2 – 7

years old); c) Concrete operational stage (7-12 years old); d) formal operational stage (13-14 years old or 14-15 years old).
In order to overcome difficulties of implementaion of depth observations and interviews for a number of students in shorter time at once, Capie & Tobin (Sumarmo, 1987b)developed a written tes called test of logical thinking (TOLT) that compiled based on theory of Inhelder and Piaget child cognitive develpoment. The TOLT was arranged in two models (A and B) and each test was in multiple choice form of five option accompanied with 5 choices of reason. The tests measured four reasoning abilities those were: controlling variables reasoning, proportional reasoning, probabilistics reasoning, and combinatorial reasoning. Capie & Tobin (Sumarmo, 1987b)arranged TOLT as group test that measured formal thinking ability for a number of students of various age and cultures in shorter time at once.

By implementing TOLT, Capie & Tobin, (Sumarmo, 1987b)excecuted cross-studies with various groups of students from sixth grade to college students of different culture with culture of Piaget's subjects. The cross studies detected there were many students of more than 15 years old did not attain formal operational stage yet (Table 1). These findings were different with the first hypothesis of Inhelder and Piaget (Sumarmo, 1987b)that subject will attain formal operational stage at 12 – 15 years old.

In addition, the studies also invented that in higher school's grade there found greater precentage of formal stage students and smaller percentage concrete students. These findings were firm with Piaget's hypothesis that child cognitive development was influenced by maturity, balanced, socialization, and transmision education and culture of the child. Related to those findings, then Piaget (Sumarmo, 1987b) proposed further hypothesis that normal subject will attain formal operational stage in between 11- 12 years old up to 14 – 15 years old, even in other condition in 18 – 20 years old.

Table 1. Findings in Validation Studies on TOLT of Capie and Tobin (1981)

Subject	N	% Concrete Stage	% Transition Stage	% Formal Stage
6th grade students	100	97	3	0
7 th grade students	86	96	2	2
8 th grade students	167	86	9	5
11 th grade students	82	32	17	51
College students	247	45	18	37

Source: Tobin and Capie (1981, as cite in Sumarmo, 1987)

Mathematical Creative Thinking and Habits of Mind

Some experts (Martin, 2009)defined creative thinking term differently, however they include four main similar components namely: fluency, flexibility, originality, and elaboration. Semiawan (Sumarmo 1987a)posed that to think creatively as to compile new idea and to apply it in solving problem, and to identify association between unclear ideas. Munandar (Sumarmo, 1987a) detailed components of creative thinking as follow: a) Fluency included: to arouse many ideas, answer, solutions, or questions; to pose many strategies, to consider more than one strategy and its result: b) Flexibility holded: to generate various ideas, answers, or questions, to percieve a problem form different poin of view; to look for different alternatives, to alter an approach or way of thinking; c) Originalityare: to bring in a new or unic idea, to consider an unusual way;to compile unusual combination from its parts or elements; d) Elaboration consists: to broaden and to improve an idea or a product; to enlarge or to specify an object, idea, or situation so that became more interesting.

Other expert, Pehkonen (Pehnoken, 1997) stated creativity as individual's ability to generate something new or unpredictable idea. (Musbikin, 2006)clarified to think creatively as to iniciate a new idea, to develop a new relationship or unpredictable relationship, to compile non memorized concept, to invent a new solution for previous problem, and to offer a new question. Then, Martin(Martin 2009)states that to think creatively as to creat a new idea or a new way in producing a product.Other explanation of creative thinking is proposed by Puccio and Murdock (Costa, 2001a) and Balka (Mann, 2005). According to Puccio and Murdock (Costa, 2001b), creative thinking consists of cognitive, affective, and metacognitive abilities. The cognitive and metacognitive abilities contained some activities namely: to recognize a problem and an opportunity, to creat excelent and different questions, to indicate relevant and irrelevant data, problem and opportunity; to produce many ideas, many different ideas, to inspect and to assess relationship between choices and alternatives, to reform the old of thinking mode and habit, to formulate a new relationship, to broaden and to renew a plan or ideas.

Rather similar to Puccio and Murdock's opinion, Balka (Mann, 2005)specified that MCT loaded convergen and divergen thinking as follow: a) to generate hypothesis, b) to decide pattern of mathematical situation; c) to terminate a deadlock thinking by pose a new solution; d) to initiate unsual ideas and to assess its effect; e) to recognize missing information from a given problem; f) to specify general problem into its sub-problems.

To think over activities or processes on MCT, they ilustrated that MCT was classified as high order thinking (HOT) mathematics skill. That statement implied that for excecuting MCT tasks, beside should master the mathematics content student should also have a strong mathematics disposition as well, namely posessed habit to work hard and manage his or her learning habit, to be tolerant toward uncertanty, open-minded, having self confident, self controll, and unafraid to take risk on his decision. Costa names that strong disposition as habits of mind (HoM). Further Costa (Costa, 2001b) identifies sixteen indicators of HoM among other are: 1. To be resistent or impregnable; 2. To manage self concience, to think reflectively; 3. To listen to other opinion with emphaty, 4. To think flexibly and metacognitive thinking; 5. To work accurately, 6. To ask and pose questions effectively; 7. To take advantages of previous experiences and senses; 8. To think and to communicate clearly; 9. To create and to innovate; 10. To be enthusiastic on responding; 11. Unaffraid to take responsibly and to take risks; 12. Humoris, to work together, and learning continuously.

Relevent Studies

Beside the afformentioned studies' findings, by implementing original TOLT some studies reported advantages of formal stage students than concrete stage students on completing some HOT mathematics tasks. For example, McDonald and Sheehan (Sumarmo, 1987b) on structure of geometry of tenth grade students, Lawson & Lawson (Sumarmo, 1987a) on compiling arguments and testing hypothesis.

Table 2. Percentage of Concrete and Formal Stage Students Acoording to TOLT in First Pre Survey and Second Pre-survey

School Grade	Pre-survey	n	According to TOLT (IndonesiaVersion) Concrete (%)	Transition (%)	Formal (%)
Tenth Grade		46	50	35	15
Eleventh Grade (Social Science Mayor)		45	55	36	9
Eleventh Grade (Mathematics and Science Mayor)	First Pre-	97	28	19	53

Twelfth Grade (Mathematics and Science Mayor)	survey	97	10	45	45
Eleventh Grade (Mathematics and Science Mayor)	Second Pre-Srvey	92	28	19	53
Eleventh Grade (Mathematics and Science Mayor	Actual survey	414	30	22	48

Source: Sumarmo (1987), Sumarmo (2019)

In order to use TOLT on Indonesia students, (Sumarmo, 1987b) translated TOLT into Indonesian culture and validated the translated TOLT to original TOLT. Further, by implementing translated TOLT, Sumarmo (Sumarmo, 1987b) carried out two presurveys and actual survey with various groups students from tenth grade students up to twelfth grade students. Findings of Pre-survey, Actual-survey were attached in Table 2.

In addition she found advantages of formal stage students than concrete stage students on mathematical understanding and reasoning. Those findings supported that formal stage students were smarter than concrete stage students on solving high order thinking in mathematics which needed formal thinking process.

Further, by implementing Indonesia version of TOLT, Sumarmo (Sumarmo, 1987b) reported excelences of formal stage students than concrete stage students on mathematical understanding and reasoning. Those findings were stable to implication of Piaget's theory that formal stage students posses higher capabities than concrete stage students on excecuting mathematical tasks which requiring formal operational abilities.

Other recent studies Damayanti, Sumarmo, and Maya (2018) by implementing various teaching approaches invented that students reached MCT at medium to fairly good grade level. Likewise, some lately studies (Firdaus 2016)detected that students attained HoM at fairly good grade level. Seemingly, many students experienced more difficulties on accomplishing MCT tasks than on performing HoM attitudes.

RESULTS AND DISCUSSION

Results

By using Indonesia version TOLT, the survey invented percentage of students in each cognitive stage of tenth grade students of 16.25 years old were as follows: 44 % students were still at concrete stage, 19% students were at transition stage, and 36% students had reached at formal stage. These findings almost similar to finding of Capie & Tobin, (Sumarmo, 1987b) that about 53 % students of eleventh grade had already reached formal stage. Further, attaiment of mathematical creative thinking (MCT) and habits of mind (HoM) in mathematics in entirely students and according to student's cognitive stage were attached in Table 4.

Table 4. Description of Student's Mathematical Creative Thinking and Student's Habits of Mind Based on Student's Cognitive Stage

Vari-Ables	Statistics	Concrete Stage	n (%)	Transition Stage	n (%)	Formal Stage	n (%)	Total	n
MCT (IS =47)	$\bar{x}$	5.81	16	18.57	7	31.54	13	17.58	36
	% IS	12%	45%	40%	19%	67%		37%	

	SD	4.87		2.37		7.48	36%	12.89	100%
HoM (IS=120)	$\bar{x}$	92.56		97.57		92.77		93.61	
	% IS	77%	16 44%	81%	7 19%	77%	13 36%	78%	36 100%
	SD	8.61		4.72		10.30		8.71	

The survey found, entirely students attained MCT grade at very low level. These finidngs were different with findings of some recent studies Sauri (2010) that by implementing various innovative teaching approaches detected student's MCT at medium to fairly good grade level. Even if, formal students obtained MCT grade higher than transition and concrete students; formal students, transition students, and concrete students attained MCT at medium, low, and very low grade levels successively. Findings on student's difficulties on completing MCT tasks, were ilustrated in Table 5.

Table 5. Mean Score of Each Item of Mathematical Creative Thinking of Students

Cognitive Stage	Desc. Stat. / Ideal score	No.1 8	No 2. 12	No.3 12	No.4 15
Formal	$\bar{x}$	5.54	8.54	9.31	8.15
(n=13)	% out of IS	69.25%	67.17%	77.58%	54.33%
Transition	$\bar{x}$	5.14	4.57	5.86	3.00
(n= 7)	% out of IS	64.25%	38.08%	48.83%	20%
Concrete	$\bar{x}$	3.00	.88	.94	1.00
(n =16)	% out of IS	37.50%	7.33%	7.83%	.67%
Total	$\bar{x}$	4.33	4.36	4.92	3,97
(n= 36)	% out of IS	54.13%	35.83%	41%	26.47%

Formal students realized litle difficulty only on elaboration task, transition students experienced difficulties on flexibility, originality, and elaboration tasks, and concrete students realized difficulties on all items of mathematics creative tasks.

However, there were no different findings on HoM student's grades. Either in entirely or on each cognitive stage, students attained HoM at fairly good to good grade level. These findings were similar to findings of lately studies Wardani, Sumarmo, and Nishitani (2011) that found student's HoM were at fairly good grade level. Testing hypothesis of student's MCT and HoM grades among cognitive stage were attached in Table 6.

Table 6. Testing Hypothesis of Mean Difference of Mathematical Creative Thinking and Habits of Mind Among Cognitive Stages

Variables	Cog. Stage	$\bar{x}$	S	n	Sig. (1-tailed)	Interpretation
MCT	Formal	31,54	7,48	13	.00 < .05	$MCT_F > MCT_T$
	Transition	18,57	2,37	7		
	Transition	18,57	2,37	7	.01 < .05	$MCT_T > MCT_C$
	Concrete	5,81	4,87	16		
HoM	Formal	92,77	10,30	13	.161 > .05	No different HoM_F and HoM_T
	Transition	97,57	4,72	7		
	Transition	97,57	4,72	7	.83 > .05	No different HoM_T and HoM_C
	Concrete	92,56	8,61	16		

Note: MCT: Mathematical Creative Thinking Ideal score: 47
HoM: Habits of Mind Ideal score HoM :120

Further analysis, were concerning association among cognitive stage, MCT and HoM. That association were analyzed by using contigency table such as in Table 7, Table 8 and Table 9 and hypothesis testing of association using χ^2 testing were attached in Table 10.

Table 7. Contigency Between Cognitive Stage (CSt) and MCT

MCT \ CSt	Formal Stage	Trans. Stage	Concr. Stage	Total
High	13	0	0	13
Medium	0	7	0	7
Low	0	0	16	16
Total	13	7	16	36

Table 8.Contigency Between Cognitive Stage HoM

HoM \ CSt	Formal Stage	Trans. Stage	Concr. Stage	Total
High	4	3	4	11
Medium	4	4	6	14
Low	5	0	6	11
Total	13	7	16	36

Table 9.Contigency Between MCT and HoM

MCT \ HoM	High	Medium	Low	Total
High	8	5	0	13
Medium	0	7	0	7
Low	0	7	9	16
Total	8	19	9	36

Table 10. Test of Pearson-Chi Square and Contigency Coefficient among Cognitive Stage, Mathematical Creative Thinking and Habits of Mind

Variables	Pearson-Chi Square (χ^2)	Df	Contigency Cofficient (C)	Sig. (1-tailed)	Interpretation
Cog.Stage and MCT	68.000[a]	4	.816	.000 < .05	High Association
Cog.Stage and HoM	3.678[a]	4	.312	.225 > .05	No Association
MCT and HoM	3.846[a]	4	.331	.140 > .05	No Association

Discussion

These findings was similar to findings of other studies that there were no association between MCT with various mathematics soft-skills. But, these findings were different with findings of some other studies that there were association between MCT with mathematics sof-skills. Those findings pointed out that there were inconsistents findings of existence of association between mathematics hard-skills and soft-skills.

CONCLUSION

Based on findings and discussion, the survey derived conclusions as follows:
By using TOLT, there were many tenth grade students of 16.25 years old had not reached formal. Percentage students in each cognitive stage were as follow: and 36 % formal students, 19 % transition students, and 45% concrete students. In entirely, student's mathematical creative thinking was at low grade level, however according to student's cognitive stage, formal students obtained higher grade than transition an concrte students. Those grades were mediun, low and very low grade levels successively for formal, transition, and concrete students. Likewise, transition students realized difficulties on solving flexibility, originality, and elaboration tasks, while concrete students experienced on compiling all mathematics creative tasks. However, either in entirely or according to student;s cognitive stage there were no different grades of student's HoM and those grades were at fairly good and good grade level.

Further conclussion, there was high association between cognitive stage and mathematics creative thinking, but there were no association between cognitive stage and habits of mind, and between mathematics creative thinking and habits of mind. In general, findings of this survey was fitting to further Piaget's hypothesis that normal students will reach formal operational stage in 13 -14 years old or in 14-15 years old and in certain conditon in 18-20 years old. Beside that, these findings was also conformed with Piagets conception that formal operational stage students were smarter than transition and concrete students on compiling HOT mathematics tasks that needed ormal operational thinking.Based on the conclusion the survey proposed suggestions as follow. The cognitive operational stage of students are not determined by biological age of students, but are decided on student's reasoning abilities that measured by using TOLT or other relevant instrument such as Longeot Test.

To improve student's MCT, it is suggested among other are: pose student on open-ended and ill-structured mathematics problems and asked them to accompany their solution with concepts or rules used in each step of solution process. In addition, to enchance better students' habits of mind (HoM), it was suggested as follow: Be aware of students and teacher to understand of behaving HoM attitudes and they should be accustomized to behave as wished in HoM attitudes; teacher should carry out integrated and continous mathematics teaching processtion.

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