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Using Geogebra in Teaching Plane Vector

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Abstract

The study of geometry knowledge is an important component of high school mathematics. Plane geometry is the main thread that runs through high school geometry knowledge. Vector learning is the focus and difficulty of plane geometry. When students encounter a new knowledge, it will be difficult to overcome it, and it is difficult to form a new logical system in their minds. Therefore, when students learn the content of vectors, they will have cognitive conflicts at different levels of thinking. The calculation from planar geometry to directed line segments needs a transition. It is difficult to understand the concept and nature of vectors without the help of teachers. This study attempts to guide the application of Geogebra software in teaching plane vector by using Van Hill Theory to intuitively display abstract geometric knowledge, and guide students to perform dynamic operations and deductive reasoning , Gradually help students transition from point to surface to vector operation, improve classroom teaching efficiency, and thereby optimize geometry teaching, in order to provide some reference for front-line teachers.

Keywords: Geogebra software; geometry teaching; plane vector

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INTRODUCTION

According to the survey, when middle school students are learning geometric mathematics, they will find it difficult to understand the abstraction (Badraeni et al., 2020; Dini, Wijaya, & Sugandi, 2018; Wijaya, Dewi, Fauziah, & Afrilianto, 2018). There is no framework for the knowledge in their minds and they cannot be related. Therefore, the study of geometric knowledge has become a difficult point for students to learn mathematics, and flat vectors are one of the most difficult contents for students to master (Xue Shenglin, 2020). With the rapid development of modern information technology, educational information technology has gradually entered the mathematics classroom of primary and secondary schools (Aréchiga Maravillas, Díaz, Salazar-Torres, & Andrade, 2019), which not only provides convenience for teachers' teaching (Huang, 2018; Pusparini, Riandi, & Sriyati, 2017), but also helps students understand abstract knowledge (Listiawan, Purwanto, As’Ari, & Muksar, 2018; Widowati, 2019). At present, the most widely used dynamic teaching software is the geometric drawing board (Cunhua, Ying, Qunzhuang, & Wijaya, 2019; Wijaya, Ying, & Purnama, 2020; Yi, Ying, & Wijaya, 2019), but the shortcomings in the teaching process have gradually emerged and are difficult to solve. The recently emerged Geogebra dynamic teaching software can completely replace the teaching of geometry drawing board (Kağizmanli, Tatar, & Akkaya, 2011; Medina & Valdés, 2015), and further optimize and integrate all aspects of mathematics teaching, help students visualize the abstract and difficult-to-understand geometry knowledge, and help students gradually build a knowledge framework of geometry learning (Kou Hengqing. 2015). At the same time, the high school mathematics curriculum standards (2017 version) pointed out: pay attention to the deep integration of information technology and mathematics courses to improve the effectiveness of teaching. Continuously guide students to realize the scientific value, application value, cultural value and customs value of mathematics (Ministry of Education of the People's Republic of China. 2017). It can be seen that the use of Fan Hill theory to guide the use of Geogebra software can fully optimize the teaching of geometric modules and gradually cultivate students' geometric thinking.

Geogebra was designed by Markus Hohenwarter, a professor in the Department of Mathematics, University of Atlanta, Frida, USA, in the early 21st century (Reis, 2010). It has two major functions: geometry and algebra (including calculus). It makes up for the deficiencies of other dynamic software and conforms to the requirements of the era of knowledge economy (Gong Hui. 2020). China introduced the software in 2010, and most of it is used in classroom teaching in primary and secondary schools to assist teachers in teaching. Combining the idea of combining numbers and shapes in the process of using, combining pictures and texts, dynamic display, and exploring laws, can enable students to firmly grasp the knowledge of geometry, deeply understand the subtleties of mathematics, and improve geometric thinking, becoming a powerful assistant for teachers. Therefore, Geogebra software has been favored by many teachers.

In the 1950s, the Dutch scholars Van Hill and his wife divided geometric thinking into the following five levels based on Piaget ’s cognitive theory (Zhang Zhiyong. 2018):

Level 1 (Vusualization): Students can get a preliminary understanding of the figure through the outline of the figure.

Level 2 (Analysis): The student has been able to recognize the characteristic nature of the graph and will analyze the constituent elements of the graph.

Level 3 (informal deduction): Students have been able to form abstract definitions, distinguish necessary and sufficient conditions for concepts, and can classify graphics through informal reasoning, that is, non-strict proof.

Level 4 (Formal deduction): Students can understand the axioms, definitions, theorems, etc. in geometry, and they can also infer new theorems and establish a network of relationships between theorems.

Level 5 (Rigor): Students can rigorously establish theorems under different axiom systems to analyze and compare different geometric systems, such as the comparison between European-style geometry and non-European geometry.

METHOD

Design Concept

A qualitative case study approach used to explore how the lesson study process can be instrumental in equipping teachers in developing their question as part of the improvements on classroom discourse, teacher instructional practices and student cognitive engagement in this study. About the teacher’s question and student cognitive engagement and revise the lesson design. This lesson is selected from the 2.3.4 section of Chapter 2 of Mathematics Compulsory IV of the People's Education Edition in China, and the content to be learned after learning the coordinate operation of plane vectors. Students have mastered the basic concepts of plane vector theorem, orthogonal decomposition, coordinate representation, etc., and have a certain understanding of vector operation. The learning of this lesson is the deepening of collinear vectors. The use of Geogebra software for auxiliary teaching is helpful for students to distinguish the essential content of different knowledge points, gradually derive the midpoint coordinate formula, and deepen the study of plane vectors, which is the number product of plane vectors. Learn to make a bedding (Li Lan, Cao Yujia, Guo Mingjuan, Bao Yafei. 2018). According to the nature of the inquiry class and the teaching model given in this study, this lesson can be divided into five links: situation introduction, inspire interest; use software to explore new knowledge; skillful use of technology, deductive reasoning; variant application, expand new knowledge; framework summary, improve thinking.

Fan Hill's theory has the characteristics of order, advancement, and discontinuity. The transition from one level to another is uneven, requiring a "jumping" process, in which a "thinking crisis" needs to be experienced. In order to help students complete the jump between successive levels, which coincides with the design concept of this lesson. According to the characteristics of plane vector learning, such as challenging, open and creative, Van Hill theory can help students solve the "thinking crisis" in each process. At the same time, Fan Hill's theory of geometric thinking level division can provide an effective and feasible support for the teaching of this lesson, complete a deep understanding of knowledge and skills, and promote the gradual deepening of geometric thinking. Therefore, this study uses Fanhill theory as a support and adopts the teaching method of integrating dynamic technology to design the teaching framework as shown in Figure 1, which includes three main contents: five levels, teaching process and design intent.

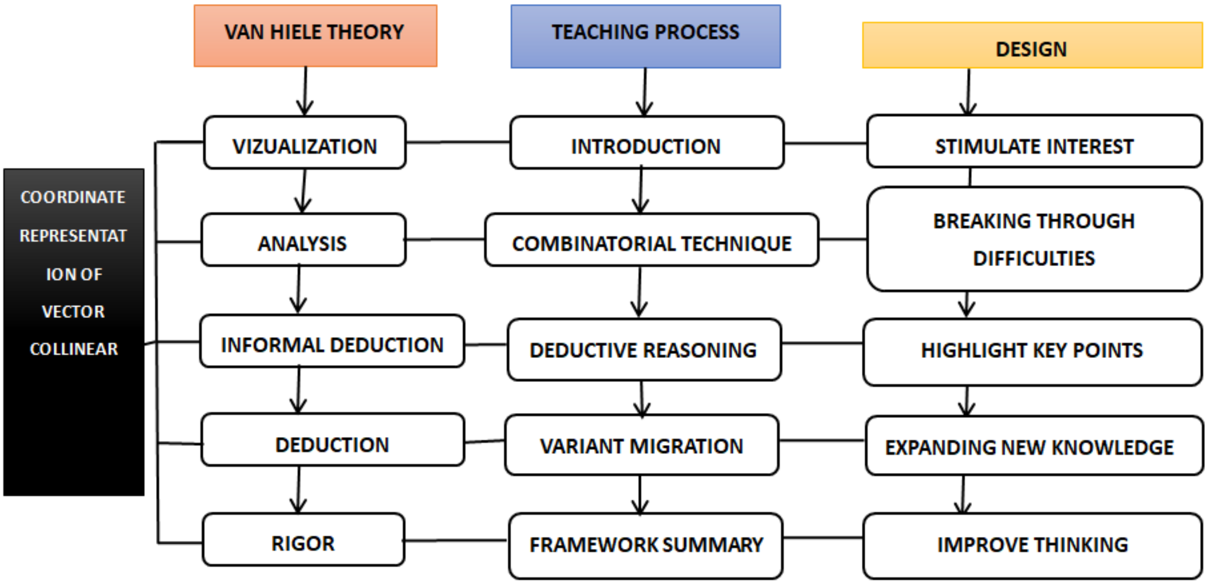

Figure 1. Framework of teaching design

RESULTS AND DISCUSSION

Teaching Process

The teacher shows the student a picture of a cube (see Fig 2), and asks the student to observe the picture with questions. After thinking, give the answer in your mind. The questions are as follows:

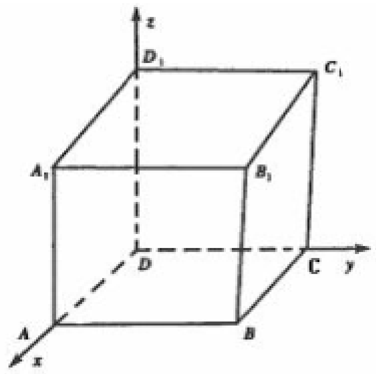

Figure 2. Vector in a cube

Question 1 : This is our common cube. Can you find those collinear vectors in the cube?

Question 2 : How many forms of representation of collinear vectors can you express the collinear vectors in the figure?

Question 3 : When two vectors are collinear, what is the relationship between their coordinates.

Create a problem situation at the beginning of teaching, let students abstract mathematical knowledge from common cubes, and transform real-world problems into mathematical problems. From three simple questions, cut into the theme of this lesson to enhance students' confidence in learning mathematics, at the same time to review the old knowledge, consolidate the purpose of learning, and pave the way for the study of the coordinates expressed by the collinear plane vector.

Use software to explore new knowledge

With the doubts of question 3, the teacher guides the students to carry out the inquiry of "coordinate representation of plane vector", and organizes the students to think independently on the draft paper. After the student's inquiry is over, the teacher uses the Geogebra software to lead the student to test the result and observe whether it is consistent with the conclusion of his own inquiry. The image drawn by the software is shown in Fig 2.

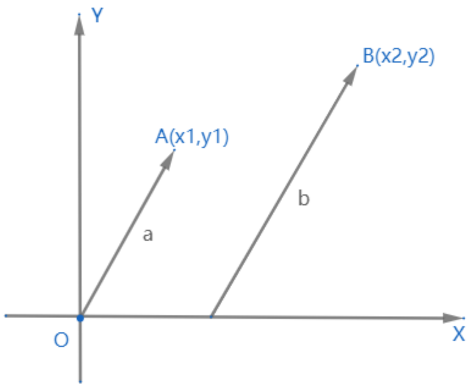

Figure 3. Coordinate representation of collinear vectors

Question 4 **:** Knowing the collinearity of two vectors $\vec{a}$、$\vec{b}$, the vector collinearity theorem $\vec{a} = \lambda \vec{b}$, how to express the vector collinearity theorem with coordinates?

Question 5 **:** It is found that the two formulas obtained after bringing in two coordinates both have λ, how to eliminate λ to get a formula containing only the horizontal and vertical coordinates?

Question 6 : Do you get $x_1y_2 - x_2y_1 = 0$ after eliminating λ, is it consistent with the results of your inquiry? This is the key content of this lesson, "Coordinate Representation of Plane Vectors."

After the students think independently, they will explore the whole class. In the process of inquiry, they will help students solve the cognitive conflicts that arise during thinking, and deepen their understanding of the vector collinearity. Through intuitive dynamic presentation, the abstract mathematical problems are visualized to help students understand the new knowledge and establish a preliminary understanding of the collinear vector coordinate representation.

Ingenious use of technology, deductive reasoning

After learning the collinear vector coordinate representation under special circumstances, and completing two simple exercises, after consolidating the new knowledge, the teacher guides the students to work in groups to discuss the coordinate representation of the collinear vector under the general situation. It also uses Geogebra software to dynamically demonstrate the general situation, let students operate it, and observe the situation when the moving point on the line segment is exactly the midpoint of the line segment, and how to express the coordinates of the point. After the discussion, use software to test the conclusion again, as shown in Figure 3.

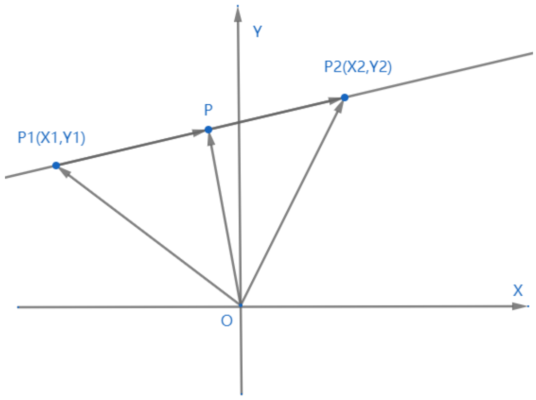

Figure 4. Vector midpoint coordinates

Question 7 **:** Let point P be a point on line segment P_1P_2. The coordinates of P_1 and P_2 are (X_1, Y_1) and (X_2, Y_2) respectively. When point P moves to the midpoint of P_1P_2, find the coordinates of point P.

Question 8 : When the point P is the trisection of the line segment P_1P_2, find the coordinates of the point P. Is the point P at this time similar to the question 7 with only one point?

Question 9 **:** Can students infer the coordinates of point P at the time of n bisector? What is the rule?

In this session, teachers seize the students 'recent development areas and use students' existing cognition to guide students to explore the general law of vector collinear coordinate representation from special to general situations with questions, which helps to develop students Geometric thinking. In the derivation process, Geogebra software is used to turn students 'existing ideas into reality, which is conducive to cultivating students' innovative thinking.

Variant application, expand new knowledge

In the application of variants, you can check whether the students have a firm grasp of what they have learned in this lesson, which is helpful for teachers to adjust classroom teaching. In the teaching of this session, the teacher connects the lessons learned in this lesson through several examples from shallow to deep, combining the students 'existing cognition to examine the students' ability to use knowledge to solve practical problems. The topics are as follows:

(1) If vectors $\vec{a} = (-1, x)$ and $\vec{b} = (-x, 2)$ are collinear and in the same direction, find the value of x.

(2) If a point P on the line passing through points $P_1(2,3)$ and $P_2(6,-2)$ makes $|\overrightarrow{PP_1}| : |\overrightarrow{PP_2}| = 3:1$, find the coordinates of point P.

(3) Inquiry question: Knowing points A (2, 3), B (5, 4), C (7, 10), if $\overrightarrow{AP} = \overrightarrow{AB} + \lambda\overrightarrow{AC} (\lambda \in R)$, ask: when λ is the value, point P is in the first and third quadrants The angle bisects the line? When λ takes a value in what range, point P is in the third quadrant?

Through three gradual thinking questions from shallow to deep, spiraling upwards, the students 'thinking is triggered, the students' thinking is developed, and the purpose of consolidating new knowledge is achieved. When explaining the last question, the teacher uses Geogebra software to discuss with the students to construct a geometric framework for the students, which helps students think, improve geometric thinking, and optimize the teaching of geometry.

Framework summary, improve thinking

With students as the main body, teachers have the highest classroom efficiency in guiding students to explore and study. In the final summary, give the class to the students, let the students take the group as a unit, use the mind map to present the knowledge learned in the whole class, and display excellent works, as shown in Figure 5.

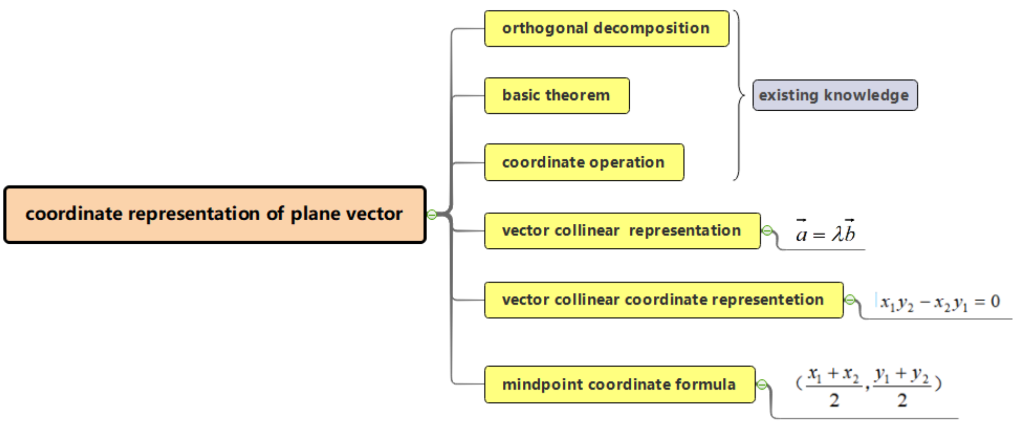

Figure 5. Mind map

Returning the classroom to the students will help them develop their innovative ability. From the drawing of the mind map, we can see how the students master the knowledge of this lesson, and what needs to be added and improved. At the same time, students can develop the habit of summarizing and reflecting, which is helpful for future study.

Discussion

In reviewing old knowledge, lay a good geometric foundation.

Concentrate the cultivation of geometric thinking in one class. Its design basis is to convert abstract geometric problems into intuitive dynamic presentations, reduce the difficulty of students 'understanding, and help students lay the foundation of geometric thinking; Dynamic operation for deep learning. For example, the "situation introduction" link, through the common cubes in life, causes students to review the knowledge they have learned, and deepens their impression of existing cognition.

Cultivate geometric thinking in dynamic demonstrations

The class revolves around the main problem of "how to represent collinear vectors", transfer knowledge to similar situations, and constantly revise students 'understanding of collinear vectors through gradual questioning and intuitive dynamic demonstrations, and gradually develop students' geometry thinking. Such as the "deductive reasoning" link, through analogy, transfer and other methods, solve the students' conflict of thinking, help students transition to the next level of thinking, and cultivate geometric thinking.

In teaching design, optimize geometry teaching

The use of dynamic technology in teaching design can optimize teaching methods, realize the transformation from traditional teaching methods to modern teaching methods, and improve the evaluation method of students by observing whether the students' performance in the classroom is in line with the preset Matching to evaluate students, provide a reference for improving teaching, with a view to optimizing geometry teaching[7] . Geogebra software is the most powerful geometry software in the world at present. It conforms to the teaching concept and can be used in teaching to make up for the shortcomings of traditional courses. For example, the "exploration of new knowledge" and "deductive reasoning" links use Geogebra software to assist teaching, helping students build geometric frameworks from scratch to deepen, which not only enriches cognition, but also facilitates the development of thinking and achieves the purpose of optimizing teaching .

CONCLUSION

Using Geogebra in Teaching Plane Vector can be used In reviewing old knowledge and lay a good geometric foundation. Geogebra software is the most powerful geometry software in the world at present. It conforms to the teaching concept and can be used in teaching to make up for the shortcomings of traditional courses. The most part of lesson study that can im Gradually help students transition from point to surface to vector operation, improve classroom teaching efficiency, and thereby optimize geometry teaching, in order to provide some reference for front-line teachers.

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