

**MATHEMATICAL REASONING ABILITY AND RESILIENCE
(Experiment with Senior High Students Using Inductive and Deductive Approach and
Based on Student's Cognitive Stage)****Siti Maesaroh¹, Utari Sumarmo², Wahyu Hidayat³**^{1,2,3}IKIP Siliwangi Bandung¹smaesaroh101@gmail.com, ²utari.sumarmo@gmail.com, ³wahyu@ikipsiliwangi.ac.idReceived: Jul 17th, 2020; Accepted: Aug 18th, 2020**Abstract**

The background of this study is that the unsatisfactory student's grade on mathematical reasoning ability (MRA) tasks. Even though MRA is an ability that needs to be possessed by high school students. To overcome this problem, we carried out an experiment for improving students' MRA and mathematical resiliency (MR) through Inductive -deductive approach (IDA) and based on student's cognitive stage (CS). The research involved 68 eleventh grade students with 16.5 years old from a high school in Bandung Indonesia. The instruments of this study were the MRA, TOLT and MR scale. The study found that as many as 29.4% student were at concrete stage, 51.5 % at transition stage, and 19.1% at formal stage. Overall and based on student's cognitive stage, the study found that the grades of MRA and MR of students who obtained IDA were better than the grades of students who taught by discovery learning approach (DLA). Based on students' cognitive stages, there was no difference grades of MRA and MR in the two teaching approaches. Besides that, study found that there was no interaction between teaching approaches and cognitive stage on student's MRA and MR and the IDA took higher ole than the cognitive stage on obtaining student's MRA, and MR. It was also found that students taught by DLA experienced more difficulty in completing MRA tasks. Besides that, there was moderate association between MRA, and MR, but there were no association between MRA and CS and between MR and CS. In addition, students performed active learning during the IDA lessons such as to discuss actively, to solve problems enthusiastically, and to present their work in front of the class voluntary.

Keyword: Mathematical Reasoning Ability, Resilience, Cognitive Stage, TOLT, Inductive-Deductive Approach

Abstrak

Latar belakang penelitian ini adalah bahwa nilai siswa yang tidak memuaskan pada tugas kemampuan penalaran matematis (MRA). Padahal MRA adalah kemampuan yang perlu dimiliki oleh siswa sekolah menengah. Untuk mengatasi masalah ini, kami melakukan percobaan untuk meningkatkan MRA dan kemampuan resiliensi matematika (MR) siswa melalui pendekatan Induktif-produktif (IDA) dan berdasarkan pada tahap kognitif siswa (CS). Penelitian ini melibatkan 68 siswa kelas sebelas dengan usia 16,5 tahun dari sebuah sekolah menengah di Bandung Indonesia. Instrumen penelitian ini adalah skala MRA, TOLT dan MR. Studi ini menemukan bahwa sebanyak 29,4% siswa berada pada tahap konkret, 51,5% pada tahap transisi, dan 19,1% pada tahap formal. Secara keseluruhan dan berdasarkan tahap kognitif siswa, penelitian ini menemukan bahwa nilai MRA dan MR siswa yang memperoleh IDA lebih baik daripada nilai siswa yang diajarkan dengan pendekatan discovery learning (DLA). Berdasarkan tahapan kognitif siswa, tidak ada perbedaan nilai MRA dan MR dalam dua pendekatan pengajaran. Selain itu, penelitian menemukan bahwa tidak ada interaksi antara pendekatan pengajaran dan tahap kognitif pada MRA dan MR siswa dan mengambil peran yang lebih tinggi daripada tahap kognitif untuk mendapatkan MRA siswa, dan MR. Juga ditemukan bahwa siswa yang diajar oleh DLA mengalami lebih banyak kesulitan dalam menyelesaikan tugas MRA. Selain itu, ada hubungan moderat antara MRA, dan MR, tetapi tidak ada hubungan antara MRA dan CS dan antara MR dan CS. Selain itu, siswa melakukan pembelajaran aktif selama pelajaran IDA seperti untuk berdiskusi secara aktif, untuk memecahkan masalah secara antusias, dan untuk mempresentasikan pekerjaan mereka di depan kelas sukarela.

Kata Kunci: Kemampuan Penalaran Matematis, Kemampuan Resiliensi, Tahap Kognitif, TOLT, Pendekatan Induktif-Deduktif

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INTRODUCTION

When we visited several mathematics lessons in one high school in 2018, we got some interesting impressions (Cahyani, Sayoga, Maesaroh, Novita, Saadah, Munawar, Fitriani, Munawaroh, Raharjo, Sumarmo, 2018). Overall, students seem to be accustomed to learning in small groups and they did not feel disturbed to complete tasks listed in the student activity sheet, even though they were being monitored by observers from outside of the school (Figure 1).



Figure 1. Students did not feel disturbed to solve derivative function problems in their group although there were observers from outside the school

Students showed their curiosity and attention in solving derivative function problems. (Figure 2).



Figure 2. Students actively completing assignments in student work sheet.

Likewise, in class discussion sessions, when teacher offered who were willing to present their group work in front of the class, many students voluntarily raised their hands (Figure 3).



Figure 3. Students raised their hands expressing their willingness to present their work in front

of the class voluntarily

Students felt able to complete the task well. Those learning environments illustrated that students having curiosity, attention and belonging ability to solve high order thinking mathematical tasks. Students showed that they had mathematical resilience as expected in the objectives of learning mathematics in affective aspects, namely: Having an attitude of appreciating the usefulness of mathematics in life, namely curiosity, attention, and interest in learning mathematics, as well as a tenacious and confident attitude in problem solving (Departemen Pendidikan Nasional, 2013). Several studies (Hendriana, Sumarmo, Carli, Ristiana, 2019, Hutauruk, Priatna, Darmayasa, 2019, Murni, & Sugandi, 2017) supported the statement that students obtained mathematical resilience (MR) at quite good grade level.

Teachers and researchers realize that MRA is a mathematical ability that needs to be mastered by high school students. There are several reasons to support the statement, namely: a. MRA is attached in the goals mathematics teaching (Departemen Pendidikan Nasional, 2013), such as: To develop students' potential to be critical, creative, logical, and conscientious individuals; b. MRA helps students not only memorize formulas, principles, and procedures for solving problems, but motivate students to apply appropriate formulas and principles in solving problems so that students reach meaningful mathematical knowledge. Several writers propose the notion of MRA in different expression, such as follow: a. MRA contains important, active and dynamic processes that needed in solving mathematics and other discipline problems (Schoenfeld, 1996); b. MRA is ability to think logically about and with mathematical objects (Brodie, 2010); c. MRA is to derive conclusions based on relevant data, event, facts, evidence, or sources (Keraf 2012, and Shurter and Pierce as cited in Hendriana, Rohaeti, Sumarmo, 2014, Shadiq, 2000).

Those notion of the MRA, illustrates that MRA contains a variety of depth of mathematical tasks ranging from low level to HOTS in mathematics. As an implication, there were diversity in student achievement in the MRA. Different findings on students' mathematical resilience (MR) had been reported, that student's MR were at pretty good grade, several studies (Aminah, Kusumah, Suryadi, and Sumarmo, 2018, Ayal, Kusumah which, Sabandar, Dahlan, 2016, Rohaeti, Budiyanto, Sumarmo, 2014, Sumarni & Sumarmo, 2017) reported that students getting treatment with ordinary teaching reached MRA at low grade level, while students taught by different innovative teaching approaches obtained MRA at low up to pretty good grades level. Those findings illustrated that MRA was difficult task to solve than to behave MR for many high school students.

As part of MRA task involves HOTS in mathematics, besides students should master the mathematics content and have a strong MR, students need to have certain prerequisite cognitive reasoning ability as well. Such kind of reasoning ability is formal reasoning stage. Inhelder and Piaget (1972, as cited in Sumarmo, 2019) by analyzing accurately the way of reasoning of various groups of children, they classified children' reasoning ability into five main gradually increasing stages namely: a) Sensory-motor stage (infant up to 2 years old); b) Pre- concrete operational stage (2 – 7 years old); c) Concrete operational stage (7-12 years old); d) formal operational stage (13-14 years old or 14-15 years old).

Regarding the learning process, Polya (1975) emphasized that the task of teacher was not merely conveying the subject matter. But the more important thing is to behave as expected by students, to encourage students to express their opinions in accordance with their own language and to help students think better. Apart from that, the Indonesian Mathematics Curriculum (Departemen Pendidikan Nasional, 2013) recommends that mathematical hardskill such as MRA and mathematics soft skills such as MR should be developed simultaneously. Based on writers' analysis on Inductive-deductive approach (IDA), we predicted that IDA will help student to improve their MRA and MR. Bruce, Weil & Calhoun (2000) propose some strategies

of inductive approach such as: a. Concept formation, b. Interpretation data; and c. Application of principle. While deductive approach is an approach which using a series of premises and logical reasoning then derive a conclusion. Two studies (Hidayat, Sabandar, Syaban, 2018, Nadia, Rohaeti, Kustiana, 2018) reported the superiority of IDA than scientific approach (SA) on improving mathematical problem solving and communication abilities and on mathematical self-efficacy.

Those afformentioned arguments and findings stimulate researchers to excecute a research to analyze the role of IDA and student's cognitive stage, on obtaining student's MCTA, and MR and then we compiled research questions as follow.

1. What are student's percentage on each cognitive stage measured by using TOLT?
2. Are MRA grade and its normalized gain, and MR grade of students getting treatment with IDA better than the grades of students taught by SA reviewed overall and based on student's cognitive stage?
3. What are student's difficulties on solving MRA tasks reviewed overall and based on student's cognitive stage?
4. Is there any interaction between teaching approaches and CS on MRA and MR?
5. Is there any association among MRA, CS, and MR?
6. What are student's activities during IDA lessons?

METHOD

The goals of this research were to analyze the role of inductive-deductive approach (IDA) and cognitive stage on students' mathematical creative thinking ability (MRA) and mathematical resilience (MR). The research involved 68 eleventh grade students, MRA test, and MR scale, and the TOLT. Except TOLT the other instruments were prepared specifically for this study and before experiment, we tried out all of the instruments. and carried out calculation of rubric scoring for each response of item of MR scale, and other description of MRA. By using Hendriana and Sumarmo (2014) and Sumarmo (2015) as references researchers obtained description of MRA test, MR scale, TOLT test were attached in Table 1.

Table 1. Description of MCTA test, MR Scale, TOLT.

Instruments	n sample	n Instrument	Reliability	Item Validity	Difficulty Index	Discriminat Power
MCTA test	59	5	.79	.70 - .92	.16 - .47	.22 - .42
MR scale	59	30	.95	-	-	-
TOLT*	92	10	.66	.42 - .84	.37 - .83	.59 - .81

Note *) adopted from Tobin and Capie (as cited in, Sumarmo (2019))

In the following we listed some sample of instruments of this study.

Sample 1. Mathematical reasoning ability test items (analogy reasoning)

Consider the case below.

The process of finding gradient of tangent line to $f(x) = x^2 + 1$ at the point $x = -2$ is similar to the process:

- 1) To find the velocity $v(t)$ to equation of motion $S(t) = 4t^3 - 5t + 12$
- 2) To find the acceleration of a when $t = 2$ to equation of motion of $S(t)$
- 3) To find the function value of $g(x)$ at point $x = 3$

- 4) To find the value of $g'(3)$ of the function g
- Choose the correct statement (more than one) from the 4 options, and write the mathematical concepts contained in the above case.
 - Explain the reasons that the other statements are false.

Sample 2. Item test of proportional reasoning of TOLT

Read carefully, and select the answer accompanied by the correct reason from the case below.

The Flower Seeds

A gardener bought a package of 21 seeds. The package contents listed:

- 3 short red flowers
- 4 short yellow flowers
- 5 short orange flowers
- 4 tall red flowers
- 2 tall yellow flowers
- 3 tall orange flowers

If just one seed is planted, what are the chances that the plant that grow will have red flowers?

Answer:

1 out of 2
 1 out of 3
 1 out of 7
 1 out of 21
 Other

Reasons:

One seed has to be chosen from among those that grow red, yellow, or orange flowers
 $\frac{1}{4}$ of the short and $\frac{4}{9}$ of the tall are red.
 It does not matter whether a tall or a short is picked. One red seed needs to be picked from a total of seven red seeds
 One red seed must be selected from a total of 21.
 Seven of the twenty-one seeds will produce red flowers.

Sample 3. Item of Mathematical Resiliency Scale

Statements

- I am lazy to write the formula used at each step of solving problem of derivative function
- I am happy to explain to solve difficult limit problems to other friends
- I feel bored studying derivative functions problem from various books
- I am desperately looking for relevant sources to complete derivative function task in daily life problem
- I feel challenged to solve difficult problem of application of derivative function
- I tried to find a new way when I failed to solve a limit problem
- I think to correct errors in solving function derivative problems is tiring

Note: SA: strongly agree, A: agree, DA: disagree, SDA: strongly disagree

RESULTS AND DISCUSSIONS

Results

Percentage of Students on each cognitive stage measured by using TOLT

By using TOLT the study found percentage of cognitive stage of students of this research as attached in Table 2.

Table 2. Percentage of Students in each of Cognitive Stage Determined by using TOLT

Test	n	Number of Subject					
		Concrete Stage		Transition Stage		Formal Stage	
		f	%	f	%	f	%
TOLT	68	20	29.4%	35	51,5%	13	19.1%

As in Table 2, the study found that only 29,4% eleventh grades students with 16.5 years old were at formal operational stage. This finding was lower than Sumarmo's finding (1987, as cited in Sumarmo, 2019) which she found 45% eleventh grade students were at formal operational stage, but it was similar to findings of recent studies (Gunawan et.all, 2019, Saepul, et.all, 2019) that they found about 25% up to 36% eleventh grade students were at formal operational stage.

Students' MRA and its gain (N-G), and MR overall and based on Cognitive Stage

Overall and based on students' cognitive stage, the attainment of students' MRA and its gain (N-G), and MR were attached in Table 3 and Table 4.

Table 3. Student's MRA and Its Gain (N-G), Based Cognitive Stage in Both Teaching Approaches

Variables	CS	Stat Des c.	Inductive-deductive Approach				Discovery Learning Approach			
			Pre-test	Post-test	$\langle g \rangle$	n	Pre-test	Post-test	$\langle g \rangle$	N
MRA (IS : 52)	For. CS	\bar{x}	4.86	39.14	.73	7	6.00	38.17	.70	6
		(%)	9.34	75.27	-		11.54	37.40	-	
		s	1.68	4.02	.08		2.10	6.31	.15	
	Tr CS	\bar{x}	4.44	40.50	.76	16	4.32	34.42	.63	19
		(%)	8.53	77.88	-		8.30	66.19	-	
		s	1.93	3.98	.08		1.60	6.47	.13	
	Cr CS	\bar{x}	6.27	39.82	.77	11	6.22	31.78	.52	9
		(%)	14.06	76.57	-		11.97	61.11	-	
		s	2.15	3.84	.09		2.17	3.15	.06	
	Ove rall	\bar{x}	5.12	46.00	.74	34	5.12	34.38	.62	34
		(%)	9.84	76.92	-		9.84	66.12	-	
		s	2.07	3.86	.09		2.01	5.97	.13	

Based on Table 3, on over all students and based on students' cognitive stages, in pre-test there were no different students' grades of MRA of both teaching approaches and the grades were at low level. These finding were rational caused of students had not learned yet the mathematics contents. But afther teaching approaches, the research found that on MRA and its N Gain, and MR, student getting treatment with IDA obtained better grades than the grades of student taught by demonstration learning approach (DLA). Students' MRA of IDA class were classified at good grades level, while in DLA class students' MRA were at moderate grade level.

As well as students getting treatment with IDA obtained MR at fairly good grade level while students taught by DLA attained MR at moderate grade level. For the whole students testing hypothesis of those mean differences of MRA, its N Gain, and MR were attached in Table 5.

Table 4. Student’s MSRL Based Cognitive Stage in Both Teaching Approaches

Variables	CS	Stat .Des c.	Inductive-deductive Approach				Inductive-deductive Approach			
			Pre-test	Post-test	$\langle g \rangle$	n	Pre-test	Post-test	$\langle g \rangle$	n
MR (IS: 159)	For. CS	\bar{x}		108.00			99.00			
		(%)		72.48	7		66.44		6	
	s			8.54			11.70			
				109.81			93.53			
	Tr CS	\bar{x}		109.81			93.53			
		(%)		73.70	16		62.77		19	
s			8.97			8.21				
			108.83			98.50				
Cr CS	\bar{x}		108.83			98.50				
	(%)		73.03	11		69.10		9		
s			13.39			5.17				
			109.12			95.29				
Ove rall	\bar{x}		109.12			95.29				
	(%)		73.23	34		63.96		34		
s			10.23			8.05				

Note:

MCTA: mathematical creative thinking ability,
MR: mathematical resilience

Ideal Score: 55
Ideal score:159

Table 5. Testing Hypotesis of Mean Difference of Mathematical Reasoning Ability (MRA) Its N-Gain, and Mathematical Resilience (MR) on the Both Teaching Approaches

Variables	Teaching approach	\bar{x}	SD	n	Sig.	Interpretation
MRA	IDA.	46.00	46.00	34	.00 < .05	MRA _{IDA} > MRA _{DL}
	DL	34.38	5.97	34		
N-Gain of MRA	IDA.	.74	.09	34	.00 < .05	N-Gain MRA _{IDA} > N-Gain MRA _{DL}
	DL	.62	.13	34		
MR	IDA	109.15	10.77	34	.00 < .05	MR _{IDA} > MR _{DL}
	DL	95.29	8.05	34		

Note: MCTA : mathematical creative thinking ability
MR : mathematical resilience

Ideal score MCTA: 55
Ideal score MR :159

Findings on MRA of this study that was at good grade level was different with findings of previous study (Aminah et.all, 2017, Gunawan, et.all, 2019) that found students obtained MRA at low-moderate grade level. However, finding of this study was almost similar to other studies’ finding (Ayal, et.all, 2016, Rohaeti, et.all, 2014, Sumarni & Sumarmo, 2017, Bernard, & Rohaeti, 2016, Mulyana & Hendriana, 2015, Maryam 2018, Napitulu, 2017) which students obtained MRA at pretty good grade level. Those findings illustrated that students obtained MRA at variety grades level.

The study findings on MR of students getting treatment with IDA that was pretty good grade level, were similar to previous studies findings (Ariyanto, et.all 2017, Hendriana, et.all, 2019, Hutauruk, & Priatna, 2017, Hutauruk, Priatna, & Darmayasa, 2019, Murni, & Sugandi, 2017) that found students getting treatment with innovative teaching approaches obtained MR at pretty good to good grade qualification.

Further analysis was testing hypothesis of MRA, N-Gain of MRA, and MR of students getting treatment with IDA based on cognitive stage, and the result were attached in Table 6. Based on the cognitive stage of students, this study detected that there were no different grades of MRA,

and MR among students of formal, transition, and concrete cognitive stage. These findings were different with findings of Sumarmo (1987, as cited in Sumarmo, 2019) that formal students achieved higher grades in mathematical reasoning than the grade of concrete operational stage student.

Table 5. Testing Hypotesis of Mean Difference of Mathematical Reasoning Ability (MRA) Its N-Gain, and Mathematical Resilience (MR) based on Cognitive Stage in IDA Class

Variable	Cognitive Stage	\bar{x}	SD	n	Sig.	Interpretation
MRA	Formal	39.14	4.02	7	.419 >	No different MRA_F and MRA_T
	Transition	40.50	3.98	16	.05	
	Transition	40.50	3.98	16	.104 >	No different MRA_T and MRA_C
	Concrete	39.82	3.84	11	.05	
N-Gain of MRA	Formal	.73	.08	7	.419 >	No different N-Gain MRA_F and N-Gain MRA_T
	Transisition	.76	.08	16	.05	
	Transisition	.76	.08	16	.079 >	No different N-Gain MRA_T and N-Gain MRA_C
	Concrete	.77	.09	11	.05	
MR	Formal	106.86	8.30	7	.41 >	No different MR_F and MR_T
	Transition	109.56	9.87	16	.05	
	Transition	109.56	9.87	16	.49 > .05	No different MR_T and MR_C
	Concrete	110.00	13.81	11		

Note: MCTA : mathematical creative thinking ability
 MR : mathematical resilience

Ideal score MCTA: 55
 Ideal score MR :149

Students’ Difficulties on Solving MRA Tasks

The next analysis was about students’ difficulties on solving MRA tasks based on teaching approaches as attached in Table 6.

Table 6. Mean Score Of Each Item of Mathematical Reasoning Ability Test of Students In Both Teaching Approaches

Teaching Approach	Stat.Desc	No.1	No.2.	No.3	No.4	No.5
	Ideal score	10	10	10	12	10
IDA	\bar{x}	7,82	8,12	7,88	7,53	8,65
	% out of IS	78.20	81.20	76.80	62,75	86.50
DL	\bar{x}	5.24	7.94	6.38	7.62	7.62
	% out of IS	52.40	79.40	63.80	63.5	76.20

The result of analysis pointed out that students taught by DLA encountered difficulty on solving to determine extreme of quadratic function and write the rule used in each step of calculation tasks, while students getting treatment with IDA did not encounter any difficulty.

Interaction between Teaching Approaches and Cognitive Stage on MRA and MR

Interaction between teaching approaches and cognitive stage on MRA and MR were analyzed by using Two Path Analysis and SPSS software as in Table 7 and Table 8.

Table 7. Testing Two Path Analysis MRA Based on Teaching Approaches and Cognitive Stage

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	700.708 ^a	5	140.142	5.770	.000
Intercept	79107.919	1	79107.919	3257.184	.000
Teaching approaches	350.442	1	350.442	14.429	.000
Cognitive stage	81.246	2	40.623	1.673	.196
Teaching approaches * Cognitive stage	92.615	2	46.307	1.907	.157
Error	1505.807	62	24.287		
Total	96263.000	68			
Corrected Total	2206.515	67			

a. R Squared = .333 (Adjusted R Squared = .279)

Table 8. Testing Two Path Analysis MR Based on Teaching Approaches and Cognitive Stage

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	3516.947 ^a	5	703.389	8.188	.000
Intercept	597515.457	1	597515.457	6955.459	.000
Teaching approaches	2147.181	1	2147.181	24.995	.000
Cognitive stage	13.565	2	6.783	.079	.924
Teaching approaches * Cognitive stage	255.909	2	127.954	1.489	.233
Error	5326.170	62	85.906		
Total	719174.000	68			
Corrected Total	8843.118	67			

a. R Squared = .398 (Adjusted R Squared = .349)

Based on Table 7 and Table 8 the study found there were no interaction between teaching approaches and cognitive stage on MRA and MR (two tailed sig .000 < .005). Those findings indicated that the IDA have greater role than the role of cognitive stage on improving student's MRA and MR. This statement was in accordance as well with findings that concrete and transitional cognitive stage students getting treatment with IDA obtained higher grades than the grade formal cognitive stage students taught by DLA (Table 3).

Association among MRA, CS, and MR

Further analysis was about association among MRA, MR and CS. By using: contingency table between MRA and CS (Table 9), between MR and CS (Table 10) and between MR and CS (Table 11), statistic Pearson-Chi Square (χ^2) and SPSS for window, the research found the value of χ^2 and C coefficient and Q coefficient as in Table 12.

Table 9. Contingency between MRA and CS

MRA	CS			Total
	F	T	C	
High	5	8	4	17
Medium	2	8	6	16
Low	0	0	1	1
Total	7	16	11	34

Table 10. Contingency between MR and CS

MRA	CS			Total
	F	T	C	
High	5	10	2	17
Medium	2	6	6	14
Low	0	1	2	3
Total	7	17	10	34

*Note. F: Formal, T: Transitional, C: Concrete

Table 11. Contingency between MRA and MR

MRA	MR			Total
	High	Medium	Low	
High	14	3	0	17
Medium	3	11	2	16
Low	0	0	1	1
Total	7	14	3	34

Table 12. Test of Pearson-Chi Square and Contingency Coefficient between MRA, MR and CS, in IDA Class

Variables	Pearson-Chi Square (χ^2)	DF	Sign 2 tailed	Sign 1 tailed	Interpretation
MRA - CS	4.737 ^a	4	.315	.157 >.005	No association between MRA - CS
MRA- MR	24.002 ^a	4	.038	.019 <.005	There was association between MRA and MR
MR -CS	3.616 ^a	4	.260	.130 >.005	No association between MR - CS

Based on the analysis result listed in Table 12 the study obtained the following interpretation: a. There was moderate association students' MRA and MR with $C = .350$ or $Q = .584$; and b. There were no association between MRA and CS and between MR and CS

Those findings on association among MRA, CS, and MR were different with findings of Gunawan, et.all, (2019) and Saepul et.all (2019) that there were high association between mathematical hard skill and CS, and that there were no association between mathematical hard skills and soft skills.

Students Activities during IDA Lessons

The next analysis was about students' activities during IDA lesson. Based on observation during IDA lessons, we obtained an overview of student learning activities as follow. During IDA lessons, students seemed active learning for example they discussed in small groups to observe the presentation in Student work Sheet (Figure 4.a.), they try to understand the meaning of the derivative of the function and its formulas and they practiced problems enthusiastically (Figure 4.b.), and then they explained their work in front of the class voluntarily (Figure 4.c.).



Figure 4. Students' activities during IDA lesson

Discussions

Mathematical Reasoning Ability, Mathematical Resiliency, and Cognitive Stage

In addition to arguments on mathematical reasoning ability (MRA) had been reported, in the following we proposed argument on MRA broader. Referring to the notion of MRA several authors (Keraf 2012, and Shurter and Pierce as cited in Hendriana, Rohaeti, Sumarmo, 2017, Shadiq, 2000) and based on the way of deriving conclusion, Sumarmo (as cited in Hendriana, et.all, 2017) differentiated two kinds of reasoning namely inductive reasoning and deductive reasoning. Inductive reasoning is deriving conclusion based on observed data, and it involves: a. Transductive reasoning is to apply the truth of one case to another; b. Analogical reasoning is to draw conclusions based on similarity of processes or data; c. Generalization is to derive general conclusions based on limited observed data; d. To predict a tendency; e. To arrange explanations based on patterns, data, or models f. To construct conjectures or to analyze based on existing patterns. Based on the description above, the truth of inductive reasoning is not absolute. 141

In contrast to inductive reasoning, the truth of deductive reasoning is absolute. Deductive reasoning includes a. To carry out calculations based on agreed rules; b. Logical reasoning which covers (Tobin & Capie as cited in Sumarmo, 2019): b.1. Proportional reasoning is to reason based on proportion (Leongson & Limjap, as cited in Aminah, et.all, 2018, Tobin & Capie as cited in Sumarmo, 2019); b.2. Combinatorial is to reason based on combination of some elements (Bernoulli as cited in Aminah, et.all, 2018, Tobin & Capie as cited in Sumarmo, 2019); b.3. Probabilistic reasoning is to reason based on probability of an event (Leongson and Limjap, 2003, as cited in Aminah, et.all, 2018, Tobin & Capie as cited in Sumarmo, 2019); b.4. Correlational reasoning is to reason based on correlation between different situations (Dugan, 2003, as cited in Aminah, et.all, 2018); c. To prove which covers: direct proving, indirect proving, and proving by mathematical induction).

Apart from the definition of resilience that has been described, some authors (Johnston-Wilder & Lee, 2010a, 2010b, Johnston-Wilder, Lee, Garton, Goodlad, Brindley, 2013, as cited in Sumarmo, 2015), suggest that MR is a high-quality attitude such as: confident that he will succeed if he works hard, perform tough attitude in facing learning difficulties, desire to complete, discuss, reflect on, and investigate. Based on arguments, then Sumarmo (2015) argued that with a strong MR student not only can complete the exam questions but they can apply their knowledge in other situations.

Johnston-Wilder et.all. (2013, as cited in Sumarmo, 2015) propose four components of MR, namely: a) Believe that ability of the brain can be developed; b) have an understanding of the value of mathematics; c) Understand how to work with mathematics, d) Be aware of the help of other people. Then, they suggest three ways to improve MR, those are: determine assignments that students must complete in the class, treat students as part of the environment and make sure they are involved in the activities. Other suggestion for improving MR is proposed by Lugalia, Johnston-Wilder, and Goodall (2013 namely utilizing the role of ICT in teaching-learning

process.

Child's Cognitive Development

Interested in Inhelder and Piaget's findings, Tobin and Capie (1982, as cited in Sumarmo, 2019) developed TOLT based on Inhelder and Piaget's theory as a substitute for observation and experimental techniques in determining students' cognitive stages and can be used in large numbers students and in shorter time at once. The TOLT consisted of 10 items to measure five reasoning abilities those were: controlling variables, proportional reasoning, probabilistics reasoning, correlational reasoning, and combinatorial reasoning. Further by using TOLT, Tobin and Capie (1982, as cited in Sumarmo, 2019) found that many students with more than 16 years old had not yet reached formal operational stage.

Some studies (Sumarmo, 1987 as cited in Sumarmo, 2019, Gunawan, et.all, 2019, Saepul, et.all, 2019) by using TOLT with eleventh grade students detected as much consecutively 45% (out of 414) students, 25% (out of 36) students, and 37% (out of 36) students reached the formal operational stage. Besides that, Sumarmo (as cited in Sumarmo, 2019) reported that formal operational stage students obtained mathematical reasoning higher grade than the grade of concrete operational stage students. As well, Gunawan et.all (2019) dan Saepul et.all (2019) found there were high association between cognitive stage with mathematical hard skills, and there were no association between cognitive stage and mathematical soft skills.

Inductive - Deductive Approach

Inductive approach is an approach begins with presentation of examples and through observing their characteristics then drawing conclusions. Bruce, Weil & Calhoun. (2000) propose some strategies for inductive approach such as: a. Concept formation, b. Interpretation data; and c. Application of principle. While deductive approach is an approach which using a series of premises and logical reasoning then derive a conclusion.

Combination inductive and deductive approach is an approach which begins with presentation of examples or cases, and then find a rule followed by identifying, distinguishing, generalizing, applying the rule in solving problem. Bagus (2013) proposes some steps in inductive-deductive approach that are: a. Identify accurately the concepts or principles which will be learned; b. When the concept or principles are simple and easy to understand by students, so the mathematics content is learned deductively, c. When the rules or the principles were complex, abstract, and difficult, so the mathematics content was learned inductively. To gain meaningful understanding, after the teacher sets an example, then students are motivated to arrange a problem and then solve them.

CONCLUSION

This research concluded as follows. Based on the TOLT, from 68 students aged 16.5 years old it found as many as 29.4.% of students classified at concrete operational stage, 51.5% at transition operational stage, and 19.1% at formal operational stage. Reviewed as a whole, students getting treatment with inductive-deductive approach (IDA) achieved MRA and MR at good grade level, while the second group of students attained at moderate grade level. Students taught by DLA encountered difficulty in determining extreme of quadratic function and write the rule used in each step of calculation. Based on student's cognitive stage in IDA class, there were no different grades between formal, transitional, and concrete stage students and those grades were at good level.

Other conclusion there were no interaction between teaching approaches and cognitive stage on MRA and MR. It meant that the IDA gave higher role than cognitive stage on improving student's MRA and MR, which was shown by the finding that transitional and concrete students getting treatment with IDA achieved MRA and MR at better grades than the grades MRA and

MR of formal students taught by DLA.

Also concluded that there was moderate association between MRA and MR but there were no association between MRA and CS and between MR and CS. Other than that, students performed to be comfortable learning during IDA lessons, they learned actively in their groups, solved problems enthusiastically and presented their work in front of the class voluntarily.

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