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THE ROLE OF METAPHORICAL THINKING ON STUDENT'S MATHEMATICAL REASONING ABILITY AND SELF CONFIDENCE

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Abstract

This research is a pretest-posttest experimental control group design having a goal to investigate the role of metaphorical thinking approach (MTA) toward students' mathematical reasoning ability (MRA) and self-confidence (MSC). The research involves 64 eighth grade students of a Junior High School in Garut Indonesia, an MRA test, and an MSC scale. The findings of this research were: students getting treatment with MTA reached MRA at a moderate grade level, and students taught by problem based learning (PBL) learning obtained MRA at a low grade level. However, there were no different grades on students' MSC in both teaching approaches, and those grades were at a moderate level. Students in both teaching approaches encountered difficulties in solving MRA tasks such as on analogical reasoning about determine the value of the function, solving two linear variable equations, and writing down mathematical models involved in a calculation. Besides that, there was an association between MRA and MSC, students posed positive opinions toward implementation of MTA, and they performed active learning during MTA lessons.

Keywords: mathematical reasoning ability, self-confidence, metaphorical thinking

Abstrak

Penelitian ini adalah desain kelompok kontrol eksperimental pretest-posttest yang memiliki tujuan untuk menyelidiki peran pendekatan berpikir metaforis (MTA) terhadap kemampuan penalaran matematika siswa (MRA) dan kepercayaan diri (MSC). Penelitian ini melibatkan 64 siswa kelas delapan SMP di Garut Indonesia, tes MRA, dan skala MSC. Temuan dari penelitian ini adalah: siswa yang mendapatkan pembelajaran dengan MTA mencapai MRA di tingkat kelas sedang, dan siswa yang pembelajarannya dengan pembelajaran berbasis masalah (PBL) memperoleh pembelajaran MRA di tingkat kelas rendah. Namun, tidak ada nilai yang berbeda pada MSC siswa dalam kedua pendekatan pembelajaran, dan nilai-nilai tersebut berada pada tingkat sedang. Siswa dalam kedua pendekatan pembelajaran mengalami kesulitan dalam menyelesaikan tugas MRA seperti pada penalaran analog tentang menentukan nilai fungsi, menyelesaikan persamaan linier dua variabel, dan menuliskan model matematika yang terlibat dalam perhitungan. Selain itu, ada hubungan antara MRA dan MSC, siswa mengajukan pendapat positif terhadap implementasi MTA, dan mereka melakukan pembelajaran aktif selama pelajaran MTA.

Kata Kunci: kemampuan penalaran matematis, kepercayaan diri, pemikiran metaforis

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INTRODUCTION

Based on our visits to a Junior High Schools in Garut in 2018, we obtained interesting impressions as follows (Susanti, Helmi, Resnawati, Yulianti, Hermansyah, Mellyana, Nurwahidah, 2018). In terms of student activities during the lessons as a whole students show active learning and solved problems in small groups, dare to ask, and present group assignments in front of the class (Figure 1, Figure 2, Figure 3). Those situations illustrated that students performed self-confidence in solving mathematics tasks. However, the problems discussed had not yet been classified as HOT mathematics, for example, to solve the routine problem of the linear equation of two variables $2p + q = 1$ and. $3p – q = 4$. Students have not been trained to solve problems related to mathematical reasoning ability (MRA).

Figure 1. Students accustomed to learn in small group

Figure 2. Students presented their work group in front of the class

Figure 3. Students waited for teacher's help to understand a task in SWS

Basically, MRA and MSC are essential learning outcomes that need to be owned and developed in high school students. Apart from the MRA and MSC contained in the goals of mathematics teaching, there are several opinions of authors who support the statement above. Baroody (1993, as cited in Hendriana, Rohaeti, Sumarmo, 2017) express that MRA supports student not only to learn by heart mathematics fact, rules, and principles, but MRA will help the student to reason rationally and to select proper rules and principles in solving the problem as well so that student attain meaningful mathematics knowledge.

Some authors define MRA terms in different expressions namely: a. MRA is to derive conclusions based on relevant data, events, facts, evidence, or sources (Shadiq, 2004, as cited in Kusnandi&Sumarmo, 2010); b. MRA is the capability to think logically about and with mathematics objects (Brodie, 2010, Kusnandi&Sumarmo, 2010). When we analyzed deeply mathematical processes involved in solving the MRA task, they illustrate that MRA loads higher-order thinking (HOT) skill in mathematics. They imply in solving MRA besides students should master the mathematics content, students should possess positive disposition as well, such as beliefs in his ability that he will be able to complete any difficult task. Hendriana, Rohaeti, Sumarmo, (2017) define that positive disposition as mathematical self-confidence (MSC). Later, he elaborates MSC in detail as follows. Mathematical self-confidence (MSC) is a belief or feeling toward his own ability that includes: courageous, free, responsible feeling in doing any mathematics activity, warm and polite in interacting with other people, and conscious toward his own advantages and disadvantages.

Students condition with low MRA were found in some recent studies (Aminah, Kusumah, Suryadi, Sumarmo, 2018, Ayal, Kusumah, Sabandar, Dahlan, 2016, Gunawan, Prawoto,

Sumarmo, 2019, Sumarni & Sumarmo, 2017) that reported students taught by ordinary teaching attained MRA at low-grade level, while students getting treatment with innovative teaching approaches obtained MRA at moderate-pretty good grade level. Apart from that, the illustration of student's performance which is actively learning and showing confidence in solving problems (Susanti, Helmi, Resnawati, Yulianti, Hermansyah, Mellyana, Nurwahidah, 2018) was in line with findings of some studies (Hendriana, Johanto, Sumarmo, 2018, Putra, Putri, Fitriana, Andayani. 2018, Sumarmo, Mulyani, Hidayat, 2018). The studies reported that students attained MSC at quite a good grade level.

Considering to teacher's task, Polya (1975, as cited in Sumarmo, 2019) propose that mathematics teacher's task is not only to extend mathematics content but the more important tasks are: to behave as student's wish, to appreciate student's thinking, to motivate the student to gain new knowledge, to improve student's thinking ability, to stimulate the student to think in his own manner, and to support the student to learn well. Besides that, the Indonesia mathematics curriculum, 2013, proposes that mathematics hard-skill and soft-skills such as MRA and MSC should be improved run together. Those suggestions motivate researchers to select a certain mathematics teaching approach that supports students to improve their MRA and MSC during the lesson. After researchers analyze some teaching approaches, we decide that the metaphorical thinking approach (MTA) will appropriate with Polya's and mathematics curriculum suggestions. The metaphorical thinking approach (MTA) is a teaching approach designed for improving unusual student's thinking through a new perspective. Bruce, Weil, & Calhoun, (2000, as cited in Hendriana, 2009, Hendriana, 2018) explain that MTA is a mindset in presenting and mastering abstract mathematical concepts using everyday situations by comparing two things from different sense. As MRA loads the HOT mathematics process, so in order for students able to solve the MRA task well, students should master the mathematics content, and have high self-confidence. Hendriana, Rohaeti, Hidayat (2018) reported the advantages of MTA than ordinary teaching on improving student-teacher mathematics on mathematics questioning.

Those aforementioned arguments and findings stimulate researchers to execute a study to analyze the role of MTA and student's cognitive stage, on obtaining student's MRA, and MSC and then we compile research questions as follows.

1. Are MRA grade and its normalized gain, and MSC grade of students getting treatment with MTA better than the grades of students taught by PBL?
2. What are the student's difficulties in solving MRA tasks?
3. Is there any association between MRA and MSC?
4. What are the student's activities during MTA lessons**?**

Besides the aforementioned arguments had been presented, there was author opinion that revealed the importance of MRA and MSC being developed in students. Schoenfeld (1994) proposes that MRA contains important, active, and dynamic processes that needed in solving mathematics and other discipline problems. Sumarmo (2010) classifies MRA into two kinds those are inductive and deductive mathematical reasoning. Then, Sumarmo (2010) explains inductive MRA is the ability to conclude based on observed data of a process, and she details inductive MRA into some kinds of activities, namely: a) transductive reasoning; b) Analogical reasoning; c) Generalization; d) Predicting solution or tendency; e) Giving explanation based on the model, facts, attributes, relation or pattern; f) Applying relationship of pattern for analyzing the situation, and compiling conjecture.

While deductive MRA is the ability to conclude based on proper rules. Some of the deductive MRA are abilities: a. To carry out calculation agreed to proper rules and principles; b. To reason based on the rules of inference (propositional reasoning); to examine the validity of an argument, to prove and to compile valid argument; c. To reason based on the ratio between two or more components that is proportional reasoning (Leongson & Limjap, 2003, as cited in Aminah et.all, 2018); d. To conclude based on the probability of an event (probabilistic reasoning). Another explanation of probabilistic reasoning is to determine the chance of occurrence of n objects from N objects (Leongson & Limjap 2003, as cited in Aminah et.all, 2018); e. To correlate relationships between two different situations (Dugan, 2003, as cited in Aminah et.all, 2018); f. To prove the truth of the statement directly, indirectly, or proving by mathematics induction.

When we pay deep attention to those processes involved in MRA, we get the impression there are various levels of thinking from medium to a very high level. Therefore, we have to select definite indicators fitting for certain school level of students. For example, for junior high school students, we limit indicators of MRA on analogical, generalization, and proportional reasoning, predicting, and executing enumeration based on certain rules and principles, and select the task-relevant to the learned mathematics contents.

Based on indicators of MRA, we infer that MRA contains HOT skill in mathematics which requiring students to own positive and strong mathematics disposition. Fasikhah (1994) denotes that positive and strong mathematics disposition as self-confidence in mathematics (MSC). Bandura (1997, as cited in Hendriana, Rohaeti, Sumarmo, 2017) defines that MSC is believe feeling toward self ability in uniting and mobilizing motivation and needed resources and then arousing it into proper action which fitting to task demand.

More detailed clarification of MSC is presented by Hendriana, Rohaeti, Sumarmo (2017) as follows: a. Self-confidence is a positive attitude toward himself self and extreme confidence toward anything will be done; b. The optimist is a person's attitude that points out having a good perspective toward his self and abilities; c. The objective is a person's view toward a problem suitable to the truth properly and not according to his self; d. Responsible is a person's willingness for accepting any consequence; e. Rational and realistic is a person's analysis toward a case based on rational thinking proper to the fact.

Referring to the teaching process, Fukuyama (1995, as cited in Hendriana, Rohaeti, Sumarmo, 2017) offers four suggestions to improve students' MSC, those are: a. Comprehend task should be solved well, and complete mathematics task carefully; b. Watch ways of working as a successful person; c. Look for people's support; d. Hold out toward pressure, caused by a person who has strong self-confidence once he realizes failure but then he succeeds to overcome his failures. Other authors (Kusnadi&Sumarmo, 2010, as cited in Hendriana, Rohaeti, Sumarmo, 2017) pose another suggestion as follows: a. Believe that we will succeed in our activity; b. Stop to pretend, positive thinking, avoid negative thinking such: anxiety, failure, worried feeling; c. Believe in God's Greatness; d. Employing our strength and don't worry about our weakness; e. Don't sunken in sadness to a long time, promote wish, and don't give up easily, grow spirit and motivation, and believe all maters have to be conducted, meet living reality; f. Be aware that experienced sadness is tomorrow successfulness; g. Believe that we are not alone, self-correction, and promote patience; h. Pray and devolve to God.

Hendriana (2009), clarified that metaphorical thinking as a thinking process for mastering, clarifying, and representing abstract mathematical concepts into more concrete concepts in daily life by comparing two different situations. He elaborates metaphorical thinking into some forms as follows: grounding metaphor, linking metaphors, and definitional metaphors. In

mathematics teaching, metaphorical thinking begins with presenting a model of a real situation. Then, by using metaphor students connect the mathematics concepts with prior concepts had been known in daily life, and by using metaphorical thinking students express their understanding by their own language.

Further, Hendriana (2009) proposes some steps to execute the metaphorical thinking approach as follow:1. Present a contextual situation related to mathematics content that will be learned; 2. Identified the main concepts. Students have invited to illustrate the main concept from the given contextual situation; 3. Students used a metaphor for illustrating a concept; and 4. Teacher and students discussed the base of metaphorical thinking by analyzing the reason as the background of metaphor

Apart from the study findings reported in the previous section, the following other study findings are presented below. Afrilianto, (2012) and Nurkolis, Abdul, Mutaqin, Sugandi, Wahyudin (2018) reported the superiority of MTA than ordinary teaching on improving high school student' mathematical understanding.

A lot of studies (Bernard, & Rohaeti, 2016, Koswara, Sumarmo, Kusumah, 2012, Mulyana & Hendriana, 2015, Prasetio, Sumarmo, Sugandi, 2018, Offirston. & Sumarmo, 2012, Rijaya, Sumarmo, Syaban, 2018, Rohaeti, Budiyanto, Sumarmo, 2014) reported that by using different teaching approaches students obtained MRA at moderate-pretty good grade level. As well as, some other studies (Delina, Afrilianto, Rohaeti, 2018, Dini, Wijaya, Sugandi. 2018, Fauziah, Maya, Yusnita, 2018, Nur, Rohaeti, Maya, 2018, Solihah, Hendriana, Maya, 2018) reported that students obtained MSC at pretty good grade qualification.

METHOD

This study was a pretest-postest experimental control group design having a goal to analyze the role of MTA on students' mathematical reasoning ability (MRA) and self-confidence (MSC). The study involves 64 eighth grade students, an MRA test, and an MSC scale. The MRA test consists of 7 items, and the MSC scale consisted of 37 items. Before we used the instruments, researchers consulted the instruments to two mathematics education experts for getting a decision that the instruments have sufficient content validity degree. Then by using Hendriana and Sumarmo (2014) and Sumarmo (2015) as references, researchers obtained characteristic MRA test and MSC scale as attached in table 1.

Table 1.Characteristics MRA test and MSC Scale

Instruments	N of subject	n of instrument	Reliability	Item Validity	Difficulty Index	Discriminant Power	$t_{calculation}$ t_{table} = 1.69236
MRA test	33	7	.514	.18 - .80	.05 - .65	.03 - .55	1.00 < t < 7.50
MSC scale	33	37	.821	.20 - 1.00	-	-	.70 < t < 11.50

In the following, we attached some samples of instruments of this study.

Sample 1: Analogical Reasoning

Observe this case carefully.

To determine a movement equation of a thing $h(t) = xt - yt^2$, when it is given that the height of thing after 2 seconds is 40 m and the height after 3 seconds is 45 m,

Is similar to process of

1. To determine the cutting point line $l \equiv y + 2x = 3$ and line $m \equiv y = \frac{1}{2}x + 2$;
2. To calculate the function equiation $g: x \rightarrow px + q$, when it mis given $g(3) = 5$ and $g(1) = -3$
3. To find cutting point of graph $f(x) = x^2 - 2x + 1$ with Coordinate axis;
4. To calculate the roots of equation $2x^2 - 3x = 0$
   a. Select the true answer from the 4 options above. Write the similar concept involved in those relations.
   b. Write and explain your reason that the three other options are wrong.

Table2. Sample 2: Some statements of Self Confidence Scale

No.	Statements	SA	A	DA	SDA
1.	I am challenged to determine the solution of a difficult equation system of two linear variable problems (+)				
2.	I try to correct my mistake on solving a difficult equation system of two linear variables in spite of it takes a longer time (+)				
3.	I am worried when I failed to solve the function equation which complies with a certain condition (-)				
4.	I believe that I can look for function equation based on given information (+)				
5.	I refuse to explain my work in front of the class (-)				
6.	I an uncertain toward the truth of my work on the equation system of two variables in the last examination (-)				
7.	I believe that I can explain the solution of the equation system of two variables problem in front of the class (+)				
8.	I ask for a friend's help when I realize to solve a difficult problem of the value of a function				

Note: SA: Strongly Agree; A: Agree; DA: Disagree; SDA: Strongly Disagree

RESULTS AND DISCUSSION

Results

Description of the attainment of student's MRA and its gain (<N-G> MRA), and student's MSC were attached in table 3. In the pre-test, there were no different grades of MRA between students learned by MTA and students learn by PBL and those grades were at a low level. It was rational because the students hadn't learned the mathematical content that will be studied.

Table 3. Description of Mathematical Reasoning Ability and Mathematical Self Confidence of Students in Both Teaching Approaches

Variables	Stat.	Metaphorical Thinking Approach (MTA)				Problem-Based Learning Approach (PBL)			
		Pretest	Postest	N-Gain	n	Pretest	Postest	N-Gain	n
MRA (IS= 43)	$\bar{x}$	12.28	26.00	.46	32	12.19	21.94	.33	32
	% IS	28.56	60,47	-		28.34	51.02	-	
	SD	4.137	6.638	.16		3.922	5.180	.11	
MSC Scale (IS=130)	$\bar{x}$		90.38		32		81.22		32
	% IS	-	72.88	-		-	65.50	-	
	SD		9.33				8.54		

Note: MRA: Mathematical Reasoning Ability IS: ideal score: 43

MSC: Mathematical Self Confidence IS: ideal score: 130

However, after the teaching process, on MRA, students who learned by MTA achieve a better grade than the grade of students who study with PBL. First group students obtained MRA at medium grade level, while students taught by PBL attained MRA at a low-grade level. The testing hypothesis of those grades was attached in table 4.

Table 4.Testing Hypothesis of Mean Difference of Mathematical Reasoning Ability, Its N-Gain, Mathematical Self Confidence, and on the Both Teaching Approaches

Variable	Teaching approach	$\bar{x}$	SD	n	Sig.	Interpretation
MRA	MTA	26.00	6.638	32	0.008 < 0.05	$MRA_{MTA} > MRA_{PBL}$
	PBL	21.94	5.180	32		
N-Gain of MRA	MTA	.46	.16	32	0.000 < 0.05	N-Gain MRA_{MTA} > N-Gain MRA_{PBL}
	PBL	.33	.11	32		
MSC	MTA	90.38	9.33	32	0.000 < 0.05	$MMSC_{TA-PBL} > MSC_{PBL}$
	PBL	81.22	8.54	32		

Note: MRA: Mathematical Reasoning Ability Ideal Score (IS) MRA: 43

MSC: Mathematical Self Confidence Ideal Score (IS) MSC:130

MTA: Metaphorical Thinking Approach PBL: Problem Based Learning

Both students groups still experienced many difficulties in solving MRA problems, such as on answering problems accompanied by formulas used at each step of completion, solving analogical reasoning about determining the value of the function, solving two linear variable equations, and writing down mathematical models involved in a calculation, and proportional reasoning of daily life. Those difficulties were illustrated in table 5.

Table 5. Mean Score of Each Item of Mathematical Reasoning Ability Test of Students In Both Teaching Approaches

Teaching approach	Stat.Desc / Ideal score	No.1 12	No 2. 12	No.3 10	No.4 9
MTA	$\bar{x}$	6,88	7,63	7,66	3,84
	% out of IS	57,29	63,54	76,56	42,71
PBL	$\bar{x}$	4,31	7,06	6,72	3,84
	% out of IS	35,94	58,85	67,19	42,71

Note: MTA: Metaphorical Thinking Approach IS: Ideal Score

PBL: Problem Based Learning

In further analysis, by using contigency table and statistic Pearson-Chi Square (χ^2) and data analysis using SPSS for window, the study found that $\chi^2 = 14{,}595^a$ and C= .431 or Q = . 528, with two sided sig = .006 or one sided sig = .003 < .05. (table 6 and table 7).

Table 6. Contingency Table of Mathematical Reasoning Ability (MRA) and Mathematical Self Confidence (MSC)

MRA \ MSC	High	Medium	Low	Total
High	8	8	0	16
Medium	3	22	11	36
Low	3	5	4	12
Total	14	35	15	64

Table 7. Testing Chi-Square Mathematical Reasoning Ability (MRA) and Mathematical Self Confidence (MSC)

MRA and MSC	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	14,595[a]	4	,006
Likelihood Ratio	17,623	4	,001
Linear-by-Linear Association	6,433	1	,011
N of Valid Cases	64		

Note: MRA: Mathematical Reasoning Ability

MSC: Mathematical Self Confidence

Discussion

Finding of this study that the grades of MRA_{MTA} and its <Gain> were higher than the grades MRA_{PBL} were similar to the findings of other previous studies (Kusnandi & Sumarmo, 2010, Mulyana, & Hendriana, 2015, Prasetio, Sumarmo, Sugandi, 2018, Rijaya, Sumarmo, Syaban, 2018), that by using different innovative teaching approaches students still experienced difficulties in solving some MRA tasks, and students' grades of MRA varied from medium up to fairly good level.

Those results of testing hypothesis pointed out that there was a high association (C= .431 or Q = .528) between MRA and MSC. This finding was similar to findings of other previous studies (Aminah, et.all, 2018, Hendriana, Johanto, Sumarmo, 2018, Mulyana, & Hendriana, 2015, Rohaeti, Budiyanto, Sumarmo, 2014) that there was an association between hard skills and soft skills in mathematics.

Further analysis was about students' activities during MTA lessons. Students performed good performance and participated the lessons well, such as they work together to identify the problem actively (Figure 1), they formulate and solve problem enthusiastically (Figure 2), they presented their work in front of the class voluntary. (Figure 3)

Figure 1. Students compiled metaphor and analogy

Figure 2. Students discussed their metaphor enthutiastically

Figure 3. Students presented methaphor and analogy in front of the class voluntary

Moreover, students tended to be comfortable with the implementation of MTA. Despite first-time students were confused to learn in new strategy (MTA) and to solve new kind mathematics problems but in the next sessions students accustomed to completing tasks in SWS actively. Overall students showed positive opinions on the implementation of MTA.

CONCLUSION

Based on a study finding and discussion, the study derived some conclusion as follow. The metaphorical thinking approach (MTA) took a better role than PBL on improving students' mathematical reasoning ability (MRA), its gain, and mathematical self-confidence (MSC) as well. However those both MRA grades were at a low level, and students encountered many difficulties in solving MRA tasks. Although the MSC of the first group students was better than the MSC of the second group students, both grades of MSC were at a moderate level. In the other conclusion, there was a high association between MRA and MSC, students performed intense activities during MTA, and students performed positive opinions toward the implementation of MTA.

In order for students to master MRA better, it was suggested that students should be invited to compile, to select, and to solve non-routine problems by themselves, to write principles and or rules involved in each step of completion. Then, for improving student's MSC it was suggested that students accustomed to perform self-confidence in participating in the lessons and completing difficult tasks.

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REFERENCES

Aminah, M., Kusumah, Y.S., Suryadi, D., and Sumarmo, U. (2018). "The Effect of Metacognitive Teaching and Mathematical Prior Knowledge on Mathematical Logical Thinking Ability and Self-Regulated Learning". Paper published in *International Journal of Instruction*. July, 2018. Vol. 11-number.3.

Afrilianto, M. (2012). Peningkatan Pemahaman Konsep dan Kompetensi Strategis Matematis Siswa SMP dengan Pendekatan Metaphorical Thinking. *Jurnal Infinity, 1*(2), 192-202

Ayal, C.S., Kusumah, Y.S., Sabandar, J. Dahlan, J.A. (2016). "The Enhancement of Mathematical Reasoning Ability of Junior High School Students by Applying Mind Mapping Strategy". *Journal of Education and Practice* 7 (25), 50-58, 2016

Baroody, A. J. (1993). *Problem Solving, Reasoning & Communicating, K-8: Helping Children Think Mathematically.* New York: McMillan Publishing Company.

Bernard, M. and Rohaeti, E,E. (2016). "Meningkatkan Kemampuan Penalaran dan Disposisi Matematik Siswa SMK dengan Pendekatan Kontekstual melalui Game Adobe Flash Cs 4.0.". Paper published in: *Edusentris: Jurnal Ilmu Pendidikan dan Pengajaran.*Vol.3. No.1. April. 2016, pp. 85-94.

Brodie, K. (2010). *Teaching Mathematical Reasoning in Secondary School*. New York: Springer

Bruce, J., Weil, M., & Calhoun, E. (2000). Models of teaching. *Yogyakarta: Pustaka Pelajar*.

Delina, D. Afrilianto, M., Rohaeti,EE. (2018). "Kemampuan Berpikir Kritis Matematis dan Self Confidence Siswa SMP Melalui Pendekatan Realistic Mathematic Education. *Jurnal Pembelajaran Matematika Inovatif (JPMI)*) 1 (3), 281-288, 2018.

Dini, M., Wijaya, TT., AI, Sugandi. AI. (2018). Pengaruh Self Confidence Terhadap Kemampuan Pemahaman Matematik Siswa SMP *Jurnal Silogisme: Kajian Ilmu Matematika Dan Pembelajarannya* 3 (1), 1-7, 2018

Dugan, A. (2006). Assessing the validity and reliability of a Piagetian based paper-pencil test. 2003. Thesis (Master of Education) - Wichita State University.

Fauziah, R., Maya, R., Yusnita, A.F. (2018). "Hubungan Self Confidence Terhadap Kemampuan Pemecahan Masalah Matematis Siswa SMP". *JPMI* . Vol 1, No 5 (2018) PP. 863-872

Fasikhah, S.S.(1994). *Peranan Kompetensi Sosial pada T.L. Remaja Akhir*. Tesis, Program Pascasarjana UGM Yogyakarta.

Gunawan, Prawoto, A., Sumarmo, U. (2019). "Mathematical Reasoning And Self Regulated Learning According to Student's Cognitive Stage" . *Jurnal Pembelajaran Matematika Inovatif (JPMI)* Volume 2,No.1 . 2019. Hal. 39 -52.

Hendriana, H. (2009). *Pembelajaran Dengan Pendekatan Metaphorical Thinking Untuk Meningkatkan Kemampuan Pemahaman Matematik, Komunikasi Matematik Dan Kepercayaan Diri Siswa Sekolah Menengah Pertama* (Unpbublished Doctoral dissertation, Universitas Pendidikan Indonesia).

Hendriana. H. (2018). "Pembelajaran matematika humanis dengan metapho-rical thinking untuk meningkatkan kepercayaan diri siswa". H. *Infinity Journal* 1 (1), 90-103, 2018

Hendriana, H., Rohaeti, E.E., Hidayat, W. (2018). "Metaphorical Thinking Learning And Junior High School Teachers' Mathematical Questioning Ability" *Journal on Mathematics Education* 8 (1), 55-64, 2018

Hendriana, H., Johanto, T. and Sumarmo, U. (2018*).* "The Role of Problem Based Learning to Improve Studens' Mathematical Problem Solving Ability and Self Confidence". *Journal on Mathematics Education*, 9 (2).

Hendriana, H & Sumarmo, U (2014).*Penilaian Pembelajaran Matematika*. Bandung: Penerbit PT Refika Aditama. Bandung.

Hendriana, Rohaeti, Sumarmo, 2017 *Hard Skill dan Soft Skill Mathematika.*Bandung: Penerbit PT Refika Aditama. Bandung.

Kusnandi & Sumarmo, U. (2010). "Teaching Mathematics by using Abductive-Deductive Strategy for Improving Students'Proof Construction Ability". Paper published in: *International Journal of Social and Humaniora. Alumni. Vol. 2. pp. 21-39. July-December 2010.*

Koswara, U. Sumarmo, U. Kusumah, Y.S (2012). "Meningkatkan Kemampuan Komunikasi dan Penalaran Matematis Siswa SMA melalui Pendekatan Pembelajaran Kontekstual Berbantuan Program Autograph". Paper published in: *Educationist: Jurnal kajian filosofi, teori, kualitas, dan manajemen pendidikanVol VI. No.2,* 125-131*, July 2012* .

Leongson, J. A. Limjap, A. (2003). "Assessing the mathematics achievement of college freshmen using Piaget's logical operations. In: *Hawaii International Conference On Education In Waikiki, January* 2003.

Mulyana, A & Hendriana, H. (2015). "Meningkatkan Kemampuan Penalaran Matematik dan Kemandirian Belajar Siswa SMP melalui Pembelajaran Berbasis Masalah". Paper published in *Edusentris, Jurnal Ilmu Pendidikan dan Pengajaran*, Vol. 2, No. July 2015.

Nurkolis, Abdul, J. F., Mutaqin, H., Sugandi, A.I, Wahyudin . (2018). "Effect Of Metaphorical Thinking Learning On Improving Ability Of Mathematical Understanding Viewed From Interest Level Of Student Learning." *JPMI. Volume 1 Number* 2, June 2018, pp. 204-210

Nur, A.,Rohaeti, E.E., Maya, R. (2018)."Improving The Ability Of Mathematic Connection and Self-Confidence's Student Of Junior High School Through Problem-Based Learning" *JPMI*. Volume 1 Number 3, September 2018, pp 307-311

Offirston. T. and Sumarmo, U. (2012). "Pendekatan Inkuiri Berbantuan Software Cinderella untuk Meningkatkan Kemampuan Penalaran dan Pemecahan Masalah Matematis Siswa MTs". Paper published in: *Educationist: Jurnal kajian filosofi, teori, kualitas, dan manajemen pendidikanVol VI. No.2, 101-106, July 2012.*

Peraturan Menteri Pendidikan Nasional No. 81a Tahun 2013 tanggal 27 Juni 2013 tentang Implementasi Kurikulum Standar Isi.

Polya, G. (1975). *How to Solve It. A New Aspect of Mathematical Method.* New Jersey: Princenton University Press.

Prasetio, D. A., Sumarmo, U., Sugandi, A.I. (2018). "Improving Student's Mathematical Reasoning And Self Concept By Using Reciprocal Teaching". Paper published in *JPMI, Volume* 1 Number 3, September 2018, pp. 283-294

Putra, HD. Putri, WAS, Fitriana, U., F. Andayani. F. (2018). Mathematical Problem Solving Ability And Self-Confidence Junior High School Students. *Supremum Journal of Mathematics Education* (SJME) 2 (2), 2018

Rijaya, Sumarmo, U., Syaban, M. (2018). Improving Student's Mathematical Reasoning and Self Concept by Using Reciprocal Teaching. Paper published in *Journal of Innovative Mathematics Learning* 1 (2), 86-94. 2018

Rohaeti, E.E. Budiyanto, A.M., Sumarmo, U (2014). **"**Enhancing Mathematical Logical Thinking Ability and Self Regulated Learning of Students through Problem Based Learning". Paper published in *International Journal of Education* Vol.8, No. 1. Desember 2014. pp 54-63. Graduate School, Indonesia University of Education.

Schoenfeld, A. H. (Editor) (1994). *Mathematical Thinking and Problem Solving.* Lawrence Erlbaum Associates, Publishers. Hillsdale, US

Shadiq, F. (2004). Pemecahan Masalah, Penalaran dan Komunikasi. *Yogyakarta: Departemen Pendidikan Nasional*.

Solihah, S., Hendriana, H., . Maya, R. (2018). "Enhancing The Mathematic Communication Ability and Self-confidence of Junior High School Students Using Brain-Based Learning". *Mathematics Education Journal* 2 (2), 75-82, 2018

Sumarmo, U., Mulyani, E., Hidayat, W. (2018). "Mathematical Communication Ability And Self Confidence Experiment With Eleventh Grade Students Using Scientific Approach". *Infinity*. Volume 1 Number 1, March 2018, pp19-30

Sumarni, C. and Sumarmo, U. (2017). Penalaran Matematik dan Kemandirian Belajar: Eksperimen terhadap Siswa SMP melalui Pembelajaran Generatif. Paper published in *Edusentris: Jurnal Ilmu Pendidikan dan Pengajaran.*Vol.3. No.1. April. 2017, pp. 290-

Sumarmo, U. (2010). "Berpikir dan Disposisi Matematik: Apa, Mengapa, dan Bagaimana Dikembangkan pada Peserta Didik". [Online]. in http://www.docstoc.com/docs/62326333/Pembelajaran-Matematika. [5 Maret 2012]. Paper published in Sumarmo, U. (2013) and Suryadi, D. Turmudi, Nurlaelah, E . (Editors). *Kumpulan Makalah Berpikir dan Disposisi Matematik serta Pembelajarannya.* Mathematics Deparment of Faculty Mathematics and Science Education UPI. Bandung. (2013). 109– 122.

Sumarmo, U. (2015). *Rubrik Pemberian Skor Tes Kemampuan Matematika*. Tersedia: http/www.utari-sumarmo.dosen.stkipsiliwangi.ac.id [30 Juli2016]

Sumarmo, U. (2019). Kemampuan Pemahaman dan Penalaran Matematis Dikaitkan dengan Tahap Kognitif Siswa SMA (Summary of Disertation of Utari Sumarmo, 1987, in Post graduate of IKIP Bandung. Available in Website of IKIP Siliwangi, 2020

Susanti, P., Helmi, W., Resnawati, Yulianti, Y., Hermansyah, Y., Mellyana, & Nurwadiah, E.S. (2018). *Laporan Observasi Kunjungan Kelas di SMP Negeri 2 Cilawu Garut*. Tidak Diterbitkan.