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PROOF-BASED GUIDED INQUIRY: A STRATEGY FOR IMPROVING ALGEBRAIC THINKING SKILLS IN LINEAR ALGEBRA LEARNING

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ABSTRACT

Algebraic thinking skills play an important role in mastering mathematical concepts and problem-solving; however, Linear Algebra learning still faces difficulties, particularly in understanding proofs. Therefore, a proof-based Guided Inquiry strategy is needed to help students develop deeper algebraic thinking skills. This study aims to describe the implementation of the proof-based Guided Inquiry strategy, analyze the improvement and achievement of students' algebraic thinking skills between classes using this strategy and those using conventional learning, and describe students' responses to Linear Algebra learning through this approach. The research employed a mixed-method approach with a quasi-experimental design using a non-equivalent control group. The subjects were two classes of mathematics education students, each acting as an experimental and control class. The instruments consisted of pre- and post-tests of algebraic thinking skills, implementation observation sheets, and student response questionnaires. Quantitative data were analyzed using N-Gain and mean difference tests, while qualitative data were obtained from observations and student responses. The results showed that the implementation of learning achieved an average score of 92% (very good category). N-Gain analysis revealed that the experimental class achieved a moderate improvement (0.401), higher than the control class (0.246, low category). Moreover, student responses were positive, with an average percentage of 84.8%. These findings indicate that integrating inquiry exploration with mathematical proof effectively enhances students' algebraic thinking skills and the quality of Linear Algebra learning.

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INTRODUCTION

Algebraic thinking skills play an important role in life, especially in preparing students for the next level of education or in solving various everyday problems. Therefore, students must thoroughly understand algebraic concepts because they are tools for facing life in the future (Mundy, 2000). Algebra is a topic in mathematics that has unique difficulties (Rosyidah et al., 2021). Current learning tends to focus on students' mastery of concepts, which enables them to fulfill short-term competencies, but is less effective in equipping them with the ability to solve long-term problems in life (Raharjo, 2017). According to the *National Council of Teachers of Mathematics* (NCTM), the aspects of proof and reasoning need to be integrated into mathematics learning from elementary to secondary school levels (Dewi & Dasari, 2023). This element is central to the domain of mathematics, encompassing activities such as formulating conjectures, verifying their truth, developing and evaluating proof arguments, and selecting appropriate methods of reasoning and proof (Ariati & Juandi, 2022).

The concept of Linear Algebra is often difficult for students to understand due to its complex level of abstraction. As stated by Wulandari et al. (2020:481) in their preliminary study through interviews with students taking Linear Algebra courses, the results show that students consider Linear Algebra to be a difficult course because: 1) Linear Algebra has many theorems and properties that must be memorized, and some theorems are similar or identical, causing students to confuse them; 2) Linear Algebra material is considered abstract, making it difficult to understand; 3) Linear Algebra problems are also confusing.

In the Mathematics Education Study Program at UIN Sunan Gunung Djati Bandung, many students have difficulty solving linear algebra problems, especially those related to proofs, as seen from the final exam results for that course. This is in line with the findings of a study conducted by Hanifah & Nawafilah (2021:337), which concluded that students' difficulties in the Linear Algebra course include a lack of understanding of the concept of elementary row operations and a lack of logical ability in formulating solutions to problems.

This challenge requires lecturers to help students overcome these difficulties while creating learning experiences that support the development of algebraic thinking skills. To overcome weak algebraic thinking skills, one approach that can be used is the *Guided Inquiry* learning model, which focuses on critical thinking and analysis processes to find solutions. This method shifts learning from being teacher-centered to student-centered, with the teacher acting as a facilitator. *Guided Inquiry* aims to encourage the development of students' thinking skills and trigger curiosity-based questions (Hamimi et al., 2018). The learning process includes the presentation of problems by lecturers, guidance on finding patterns from sample analyses, the development of ideas supported by lecturers, and final confirmation (Puspitasari et al., 2019). This model involves students in solving problems and building new knowledge based on concepts they already possess (Kuhlthau, 2010).

Previous studies, such as that by (Siregar, 2016), have identified common obstacles in learning proofs, but the solutions offered focus more on providing learning resources without examining innovative learning models. Studies (Rizki, 2013) (Rizqi & Hawa, 2023)on proof learning models show an increase in higher-order thinking skills, but do not utilize exploratory approaches such as guided inquiry. Research (Ellis Salsabila et al., 2015) and (Raharjo, 2017) shows the success of inductive-deductive strategies and scientific approaches in algebra learning, but has not been linked to the guided inquiry method that emphasizes active student involvement. Additionally, research Sholikhah et al. (2014) discusses guided inquiry in problem solving, but not in the context of proof or advanced algebra. Therefore, the study entitled **“Proof-Based Guided Inquiry: A Strategy for Improving Algebraic Thinking Skills in Linear Algebra Learning”** is novel because it combines guided inquiry with mathematical proof to develop students' algebraic thinking skills in the Linear Algebra course, which has not been the focus of previous studies.

The purpose of this study is to describe the implementation of the Guided Inquiry strategy based on proof in Linear Algebra learning as an effort to improve students' algebraic thinking skills. In addition, this study aims to determine the difference in the improvement of algebraic thinking skills between students who participate in learning with the Guided Inquiry strategy based on proof and students who participate in learning with conventional strategies. This study also aims to analyze the differences in the achievement of algebraic thinking skills in both groups. In addition, this study was conducted to determine how students responded to the application of the evidence-based Guided Inquiry strategy in Linear Algebra learning.

METHOD

This study uses a mixed-method approach with a concurrent embedded design, in which quantitative data is the main focus and qualitative data is used as reinforcement. The method used is a quasi-experiment with a non-equivalent control group design. The research sample consists of two classes of students from the Mathematics Education Study Program who are taking Linear Algebra courses, namely one class as the experimental group (using a proof-based Guided Inquiry strategy) and one class as the control group (using conventional learning).

The research population consisted of all students in the Mathematics Education Study Program at UIN Sunan Gunung Djati Bandung who took the Linear Algebra course in semester 3. The research sample consisted of two classes, one class as the experimental group that received learning with the Guided Inquiry strategy based on proof and one class as the control group that received conventional learning. The research data included quantitative data in the form of pretest and posttest results of algebraic thinking skills, as well as qualitative data in the form of observation sheets, response questionnaires, and student interviews. The instruments used consisted of an essay test to measure algebraic thinking skills, observation sheets on the implementation of learning, and questionnaires and interview guidelines to determine student responses. The validity of the instruments was tested through expert judgment by expert lecturers.

Data analysis techniques were carried out using quantitative and qualitative approaches. Quantitative analysis included descriptive statistics, N-Gain scores, and Independent Samples Test to determine differences in improvement and achievement of algebraic thinking skills. Meanwhile, qualitative analysis was conducted not only to examine student responses, but also to analyze students' written work based on each indicator of algebraic thinking generational, transformational, and meta-global. This analysis aimed to identify whether each indicator was achieved by students, the types of errors that appeared, and the patterns of reasoning constructed during the proof-based Guided Inquiry learning process.

RESULTS AND DISCUSSION

Results

Description of Research Results

This study was conducted in the Mathematics Education Study Program, Faculty of Tarbiyah and Teacher Training, UIN Sunan Gunung Djati Bandung, with a total population of students in the odd semester of 2024/2025 who took the Linear Algebra course, while the sample consisted of 2 classes. The research began with a pretest in the class that used the Guided Inquiry Based on Proof strategy (experimental class) and the class that used the expository strategy (control class). followed by learning activities during three meetings, and ended with a posttest in both classes and the distribution of implementation sheets to obtain data on the application of the Proof-Based Guided Inquiry Strategy and questionnaires to students to obtain data on student responses to the implementation of the Proof-Based Guided Inquiry Strategy in Linear Algebra learning. The explanation of the data collected is as follows:

Theme 1: The implementation of the Evidence-Based Guided Inquiry strategy in Linear Algebra learning

The implementation of the Guided Inquiry strategy based on proof was carried out in three meetings with material on Euclidean space, linear transformations, and matrix rows and columns. All stages of learning were observed using observation sheets for lecturer and student activities.

Table 1. Implementation of Evidence-Based Guided Inquiry Strategy

Indicator	Persentase	Categori
Preparation	100%	Very Good
Preliminary Activities	98%	Very Good
Core Activities	92%	Very Good
Closing Activities	96%	Very Good
Total	**92%**	**Very Good**

Based on the table, the implementation of the Guided Inquiry strategy based on evidence was carried out very well in all stages, with an average percentage of 92%. This shows that the learning steps, starting from preparation, introduction, core, to conclusion, can be carried out according to plan and actively involve students. To reinforce the implementation results, the following is a documentation of student and lecturer activities during the learning process using the evidence-based Guided Inquiry strategy.

In the Generational aspect, which emphasizes the construction of algebraic expressions and equations, students' solutions revealed an initial focus on the right-hand side of the equation due to its higher level of complexity. The problem was solved step by step through an algorithmic approach involving the development of algebraic objects. For instance, students decomposed the Euclidean norm, then derived the quadratic equation, and continued the process until the final expression obtained on the right-hand side was $u_1v_1 + u_2v_2 + \ldots + u_nv_n$. Subsequently, they noted that this expression was equivalent to $u \cdot v$, which represents the left-hand side. At this stage, students generalized the result and concluded that the left-hand side is equal to the right-hand side.

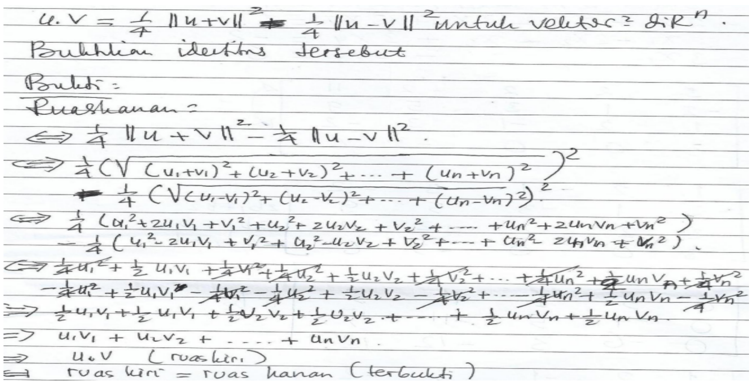

Figure 1. One of the students' answer (M1) on the sub-topic of identity/proof of vector operations on R^n.

The Transformational aspect, which involves modifying algebraic expressions or equations according to established rules, was also evident in the solution process. Finally, in the Meta-global aspect, which positions algebra as a tool not only for solving algebraic problems but also for addressing problems beyond algebra, students demonstrated its application. This was particularly visible in their manipulation of quadratic equations, which required addition and subtraction of terms within the expression.

Figure 2. Activities to Identify Problems and Formulate Hypotheses

Students collaboratively identify the problem and formulate hypotheses from the given Linear Algebra tasks, representing Steps 1 and 2 of the Proof-Based Guided Inquiry process.

Figure 3. Activities of exploring, collecting data, and verifying results

Individually, students explore and collect data as well as verify the results, which represent Steps 3 and 4 of the Proof-Based Guided Inquiry.

Thema 2: Descriptive Data of Pre-test and Post-test of Algebraic Thinking Skills in Linear Algebra Learning

The purpose of conducting pre-tests and post-tests is to determine the difference in improvement and achievement in algebraic thinking skills among students in the experimental class who received the Proof-Based Guided Inquiry strategy treatment and in the control class who received the Expository learning treatment in the Linear Algebra course. There were 32 students in the experimental class and 31 students in the control class. The following data shows the pretest and posttest results of algebraic thinking skills in the experimental and control classes, visualized in a comparison bar chart.

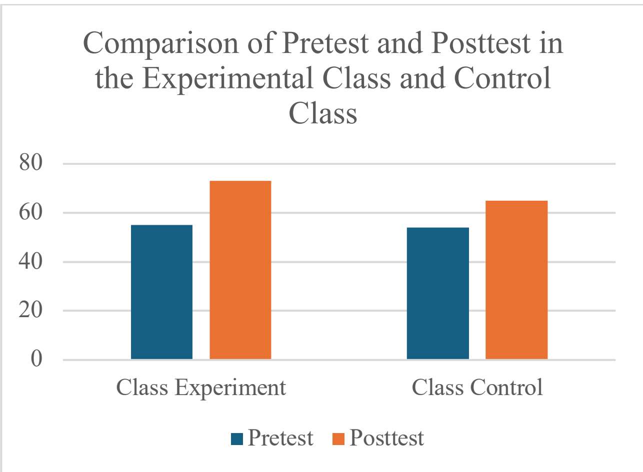

Figure 4. Diagram of comparison average improvement in pretest and posttest results between experimental class and control class

In the experimental class, which consisted of 32 students, there was an increase in scores from the pretest to the posttest for almost all students. Meanwhile, in the control class, which consisted of 31 students, there was also an increase in scores, but the average increase was not as large as in the experimental class.

Descriptive Statistics of Pre-test and Post-test Data for Linear Algebra Learning in the Experimental Class

Descriptive data from the pretest and posttest on Linear Algebra in the Experimental Class are presented in Table 2.

Table 2. Descriptive Statistics of Pretest and Posttest Data for Linear Algebra Learning in the Experimental Class

	N	Minimum	Maximum	Mean	Std. Deviation
Pretest Data	32	30.00	68.00	54.5313	12.19726
Posttest Data	32	50.00	98.00	72.8125	10.78361
Valid N (listwise)	32				

Table 2 shows that there were 32 students in the experimental class, with minimum and maximum scores on the pretest of 30 and 68 and on the posttest of 50 and 98, while the mean scores for the pretest and posttest are 54.5313 and 72.8125, respectively, and the standard deviations for the pretest and posttest are 12.1972 and 10.7936, respectively.

Descriptive Statistics Pre-test and Post-test of Linear Algebra Course in Control Class (Conventional/Expository Model)

Descriptive data for the pretest and posttest in Linear Algebra in the Control Class are presented in Table 3.

Table 3. Descriptive Statistics Pre-test and Post-test Linear Algebra Course in Conventional Classrooms

	N	Minimum	Maximum	Mean	Std. Deviation
Pretest Data	31	30.00	70.00	53.6774	10.19277
Posttest Data	31	31.00	79.00	65.0968	11.41740
Valid N (listwise)	31				

Table 3 shows that there were 31 students in the conventional class, with minimum and maximum scores on the pretest of 30 and 70 and on the posttest of 31 and 79, while the mean scores for the pretest and posttest are 53.6774 and 65.0968, respectively, and the standard deviations for the pretest and posttest are 10.1927 and 11.417, respectively.

Analysis of Algebraic Thinking Indicators

Qualitative analysis of students' written work shows that all three indicators of algebraic thinking skills were achieved by students in the experimental class. The generational indicator was met by most students, who were able to construct algebraic expressions and represent vector relationships correctly. The transformational indicator was also achieved, as students were generally able to manipulate and simplify algebraic expressions following valid algebraic rules. The meta-global indicator showed the lowest level of achievement, with some students still struggling to connect concepts or generalize results. However, the majority were able to use algebraic reasoning to solve broader problems. Overall, these findings align with the quantitative results and indicate that the Proof-Based Guided Inquiry strategy supports the development of all indicators of algebraic thinking.

Theme 3: Student Responses to Linear Algebra Learning Through Proof-Based Guided Inquiry Strategies

Student responses to the Guided Inquiry strategy were obtained from the analysis results by calculating the average response score of students. The research questionnaire was composed of two indicator aspects, namely student participation in the learning process and student attitudes toward the learning process. The questionnaire in this study consisted of 20 statements, with 10 positive statements and 10 negative statements. The following table presents the average scores of students' attitudes towards the Guided Inquiry strategy.

Table 4. Average Student Scores on Guided Inquiry Strategy

Aspect	Indicator	Average Neutral Score	Average Student Score
Student Response to Guided Inquiry Strategy	Student participation in the learning process	2,5	3,04
	Student attitudes toward the learning process		3,08

Based on Tables 4, it can be seen that the average score for each indicator is higher than the average neutral score of 2.5. From the comparison with the average neutral score, it can be concluded that the analysis of the questionnaire shows a positive response from students towards the Guided Inquiry strategy. The following are the results of the questionnaire analysis based on each indicator.

Table 5. Results of the First Indicator Response Analysis

Statement		Answer				Student Response (%)		Student Attitude Score	
No	Type	SS	S	TS	STS	Positive	Negative	Item	Mean
1	Positive	11	13	1	0	96	4	3.4	3.044
2	Negative	1	2	20	2	88	12	2.92	
3	Positive	5	19	1	0	96	4	3.16	
4	Negative	1	2	20	2	88	12	2.92	
5	Positive	8	16	1	0	96	4	3.28	
6	Negative	0	9	12	4	64	36	2.8	
7	Positive	8	17	0	0	100	0	3.32	
8	Negative	2	3	13	7	80	20	3	
9	Positive	8	14	3	0	88	12	3.2	
10	Negative	7	11	6	1	28	72	2.04	
Mean						**82,4%**	**17,8%**		

Table 5 shows the results of the analysis of student response scores on the first indicator, namely student participation in the learning process. The table shows that the average response score obtained was 3.044, meaning that students responded positively to the first indicator. Based on the average score, the percentage of positive responses from students was 82.4%, while the remaining 17.8% were negative responses. These percentages show that 82.4% of students participated in the learning process and 12.8% did not participate in the learning process.

Next, the results of the student response questionnaire to the Guided Inquiry strategy on the second indicator regarding student attitudes towards the learning process are presented.

Table 6. Results of the Second Indicator Response Analysis

Statement		Answer				Student Response (%)		Student Attitude Score	
No	**Type**	**SS**	**S**	**TS**	**STS**	**Positive**	**Negative**	**Item**	**Mean**
11	Positive	8	16	1	0	96	4	3.28	3.084
12	Negative	1	4	18	2	80	20	2.84	
13	Positive	7	16	2	0	92	8	3.2	
14	Negative	1	8	14	2	64	36	2.68	
15	Positive	5	16	4	0	84	16	3.04	
16	Negative	1	3	18	3	84	16	2.92	
17	Positive	10	15	0	0	100	0	3.4	
18	Negative	2	2	18	3	84	16	2.88	
19	Positive	12	13	0	0	100	0	3.48	
20	Negative	2	1	14	8	88	12	3.12	
Mean						**87,2%**	**12,8%**		

Table 6 presents the results of the analysis of student response scores on the second indicator, namely student attitudes towards the learning process. The table shows that the average response score obtained was 3.084, meaning that students responded positively to the second indicator. Based on the average score, the percentage of positive responses from students was 87.2%, while the remaining 12.8% were negative responses. These results show that most students have a positive attitude, such as feeling happy, interested, highly confident, and motivated in participating in the learning process, while a small number of students still show a less positive attitude.

Table 7. Recapitulation of Student Responses to the Guided Inquiry Strategy

Aspect	**Indicator**	**Percentage of Student Responses**	**Interpretation**
Student Responses to Guided Inquiry Strategies	Student participation in the learning process	82,4%	Positive
	Student attitudes toward the learning process	87,2%	Positive
Mean		**84,8%**	**Positive**

Based on Table 7, which summarizes student responses to the application of the Guided Inquiry strategy, the highest aspect obtained was student attitude in the learning process, which was 87.2% with a positive interpretation category. The aspect of student participation in the learning process was 82.4% with a positive interpretation category. The average percentage of these two aspects was 84.8%, meaning that the student response questionnaire to the Guided Inquiry strategy in Linear Algebra learning was in the positive interpretation category. The following is a pie chart illustrating the comparison between the number of positive and negative responses in the student response questionnaire to the Guided Inquiry strategy.

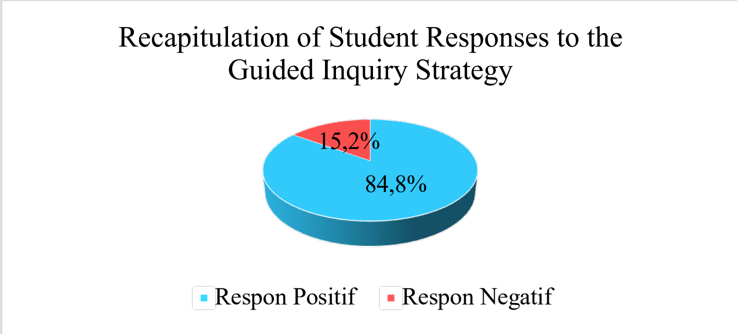

Figure 1. Recapitulation of Student Responses to the Guided Inquiry Strategy

Overall, the average response rate of students to the Guided Inquiry strategy was 84.8%, indicating that this strategy received a positive response. This percentage shows that, in general, students responded well and welcomed the learning strategy.

Hypothesis Testing

Analysis of Differences in the Improvement of Students' Algebraic Thinking Skills in Linear Algebra Learning

To determine the difference in the improvement of algebraic thinking skills between the experimental class and the control class, the first step was to calculate the normalized NGain value. This NGain value was obtained based on the results of data processing before the Proof-Based Guide Inquiry strategy was applied and data after the Proof-Based Guide Inquiry strategy was applied. This value is used to determine which improvement is better in terms of algebraic thinking skills between students who use the Proof-Based Guide Inquiry strategy learning model and those who use the conventional learning model. The processing of N-gain from both classes can be seen in Table 8.

Table 8. Results of NGain Data Calculations on Students' Algebraic Thinking Skills

Class	*N-Gain* Minimum	*N-Gain* Maksimum	Standar Deviasi	Mean *N Gain*	Criteria
Experiment	0,057	0,946	0,192	0,401	Moderate
Control	-0,031	0,577	0,163	0,246	Low

Based on Table 4.9, the average NGain value in the experimental class was 0.401 with a moderate criterion, and the average NGain value in the control class was 0.246 with a low criterion. Before testing the hypothesis, it is necessary to analyze the assumption test, namely the normality of the data and the homogeneity of the NGain data variance from the two classes above. The processing uses the SPSS version 27 program. The results of the data normality test for the NGain of the experimental class and the NGain of the control class are presented in Table 9.

Table 9. NGain Data Normality Test Results

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
N_Gain_Eksperimen	.087	31	.200*	.970	31	.511
N_Gain_Kontrol	.128	30	.200*	.964	30	.373

This output explains whether a data distribution can be considered normal or not. The decision-making guidelines are as follows :

- If the Sig value or significance or probability value is < 0.05, then the data distribution is not normal.
- If the Sig value or significance or probability value is > 0.05, then the data distribution is normal.

There are two types of data distribution normality tests that can be used, namely Komogorov-Smirnov and Shapiro-Wilk. Based on the data normality table, it can be seen that the sig value in Komogorov-Smirnov, NGain Experiment and NGain Control > 0.05 (i.e., 0.200 > 0.05), as well as in Shapiro -Wilk test, the sig values are > 0.05 (i.e., 0.511 > 0.05 and 0.373 > 0.05), so it can be concluded that the NGain Experiment and Control class scores are normally distributed.

Next, we will test the second assumption, namely the test of homogeneity of variance of the NGain data. The homogeneity test is performed to see whether the data is homogeneously varied or not. The processing uses the SPSS version 27 program, and the results of the homogeneity of variance test are presented in Table 10.

Table 10. NGain Variance Homogeneity Test Results

		Levene Statistic	df1	df2	Sig.
N_Gain	Based on Mean	81.643	1	61	.000
	Based on Median	71.313	1	61	.000
	Based on Median and with adjusted df	71.313	1	30.011	.000
	Based on trimmed mean	80.945	1	61	.000

The basis for concluding variance homogeneity is:

- If the Sig Based on Mean value is > 0.05, then the data variance is homogeneous.
- If the Sig Based on Mean value is < 0.05, then the data variance is not homogeneous.

In the Variance Homogeneity Test Results display, there is one test tool used, namely the Levene test. In the output, it can be seen that the Sig Based on Mean value is 0.000. This value is smaller than 0.05 (<0.05). This means that the two NGain data groups have different variances (not homogeneous).

Next, to test whether there is a difference in the improvement of students' algebraic thinking skills, we analyze the two NGain data above with an independent samples t-test using the SPSS version 27 program. The hypothesis proposed is :

H_0: There was no difference in the improvement of algebraic thinking skills between students who used the proof-based Guided Inquiry strategy and students who used the conventional learning model.

H_1: There is a difference in the improvement of algebraic thinking skills between students who use the Guided Inquiry strategy based on proof and students who use the conventional learning model.

The basis for testing the hypothesis is as follows:

- If the Sig value is > 0.05, then Ho is accepted.
- If the Sig value is < 0.05, then Ho is rejected.

Based on the results of the independent samples t-test analysis, a Sig value of 0.000 was obtained. This value is smaller than 0.05, so Ho is rejected and H1 is accepted. Therefore, it can be concluded that there is a difference in the improvement of algebraic thinking skills between students who use the Proof-Based Guide Inquiry strategy and students who use the conventional learning model.

Analysis of Differences in Students' Algebraic Thinking Achievement in Linear Algebra Learning

To determine the difference in algebraic thinking skills between students in the experimental class and the control class, the first step was to analyze the parametric statistical prerequisites, namely the normality and homogeneity of variance of the post-test data from both classes. Based on the analysis using SPSS version 27, the results are shown in the table below:

Table 11. Post-test Data Normality Test Results

Learning		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Skor_ Data_ Postes	Class Experiment	.099	32	.200*	.970	32	.506
	Class Control	.118	31	.200*	.917	31	.020

This output explains whether a data distribution can be considered normal or not. The decision-making guidelines are as follows

- If the Sig value or significance or probability value is < 0.05, then the data distribution is not normal.
- If the Sig value or significance or probability value is > 0.05, then the data distribution is normal.

There are two types of data distribution normality tests that can be used, namely Komogorov-Smirnov and Shapiro-Wilk. Based on the data normality table, it can be seen that the sig value in Komogorov-Smirnov, postes in the Experiment class and Control class > 0.05 (i.e., 0.200 > 0.05) only in Shapiro -Wilk test in the control class, there is a sig value < 0.05 (i.e., 0.029 < 0.05), so it can be concluded that the post-test scores in the Experiment and Control classes are normally distributed. Next, to see whether the post-test data in both classes have homogeneous variance, we can look at Table 12.

Table 12. Post-test Data Homogeneity Test Results

		Levene Statistic	df1	df2	Sig.
Skor_Data_Postest	Based on Mean	.265	1	61	.609
	Based on Median	.155	1	61	.695
	Based on Median and with adjusted df	.155	1	60.837	.695
	Based on trimmed mean	.214	1	61	.645

The basis for concluding variance homogeneity is:

- If the Sig Based on Mean value is > 0.05, then the data variance is homogeneous.
- If the Sig Based on Mean value is < 0.05, then the data variance is not homogeneous.

In the Variance Homogeneity Test Results display, there is one test tool used, namely the Levene test. In the output, it can be seen that the Sig Based on Mean value is 0.609. This value is greater than 0.05 (> 0.05). This means that the two groups of post-test data have the same variance (homogeneous).

Next, to test whether there is a difference in the achievement of students' algebraic thinking abilities, we analyze the two post-test data above with an independent samples t-test using the SPSS version 27 program. The hypothesis proposed is

H_0: There was no difference in algebraic thinking skills between students who used the proof-based Guided Inquiry strategy and students who used the conventional learning model.

H_1: There is a difference in algebraic thinking skills between students who use the Guided Inquiry strategy based on proof and students who use the conventional learning model.

The basis for testing the hypothesis is as follows:

- If the Sig value is > 0.05, then Ho is accepted.
- If the Sig value is < 0.05, then Ho is rejected.

A descriptive analysis of both data sets can be seen in Table 13.

Tabel 13. Descriptive Analysis of Post-Test Data

	Learning	N	Mean	Std. Deviation	Std. Error Mean
Skor Data	Class Experiment	32	72.8125	10.78361	1.90629
Postest	Class Control	31	65.0968	11.41740	2.05063

Based on the results of descriptive analysis, it can be seen that the average post-test score in the experimental class was 72.8125, which was higher than the average post-test score in the control class, which was 65.0968. Furthermore, based on the results of the independent samples t-test analysis, a sig value of 0.008 was obtained. This value is smaller than 0.05, so Ho is rejected and H1 is accepted, so it can be concluded that there is a difference in the achievement of algebraic thinking skills between students who use the Proof-Based Guide Inquiry strategy and students who use the conventional learning model.

Discussions

The Evidence-Based Inquiry Guide strategy applied in this study emphasizes students' creativity in actively and independently seeking solutions to problems. This study also highlights the urgency of the lecturer's role as a good and effective facilitator, providing appropriate support to students to develop their algebraic thinking skills.

Based on the results of the research that has been conducted, the implementation of the learning process with the Proof-Based Inquiry Guide strategy from the results of the data on the implementation of the Proof-Based Inquiry Guide strategy in Linear Algebra learning as a whole has been carried out very well. This can be seen from the average score for the implementation of Linear Algebra learning using the Proof-Based Inquiry Guide strategy of 92%, which is categorized as very good.

Based on the results of the implementation questionnaire, it can also be seen that the Evidence-Based Inquiry Guide strategy involves students in identifying problems, formulating hypotheses, exploring data, verifying results, and making generalizations and drawing conclusions, where these stages can train students to learn, act, and think like scientists (Hairida, 2016; Harlen et al., n.d.; Susilowati et al., 2018). In addition, Smith et al. (2007) stated that the application of Guide Inquiry shows learning activities that make students actively observe, ask questions, collect other necessary information, plan research, collect data, analyze and interpret data, provide answers, and communicate the results.

The selection of Linear Algebra material has been designed in such a way as to support the implementation of the Proof-Based Guide Inquiry strategy, especially those that can be linked to indicators of algebraic thinking skills. As stated by Susilawati et al. (2024:781), in applying the Guide Inquiry model, there are several important factors for achieving success, including the selection of relevant material, the arrangement of quality tasks, and the provision of feedback. However, according to her opinion, there are still several challenges that must be overcome in implementing the Guide Inquiry learning model, including limited resources and time, which often become obstacles for educators to implement it effectively.

Furthermore, the results of testing the hypothesis to answer the second problem formulation show that there is a difference in the increase in students' algebraic thinking skills in Linear Algebra learning. This is based on the calculation of the N-Gain test of students' algebraic thinking skills using the Proof-Based Guide Inquiry strategy compared to those using the conventional (expository) learning model, where the average N-Gain in the Proof-Based Guide Inquiry strategy class was 40.1% and the N-Gain in the control class was 24.6%. Based on the results of the N-Gain hypothesis test in each class using the Independent t-test, the average N-Gain results showed that the algebraic thinking skills of students who used the Proof-Based Guided Inquiry strategy were higher than those who used conventional learning.

This is in line with the results of research conducted by Darmawati et al. (2025:155), which concluded that the application of Guided Inquiry learning can improve students' algebraic thinking skills, with a difference in the average N-Gain score of 34.641% for the conventional class and 61.062% for the Guided Inquiry class. Another study conducted by Hutahaean et al. (2016) found that the difference in learning outcomes was due to the Guided Inquiry model in terms of knowledge because it had learning stages that improved students' knowledge.

Post-test data analysis was used to answer the third problem formulation, which was to see the difference in students' algebraic thinking skills in Linear Algebra learning by comparing the post-test results in the experimental class and the control class in both types of learning. The results of the analysis showed that the average final score in the class that used the Proof-Based Guided Inquiry strategy was higher than that in the conventional class in both Linear Algebra lessons. This indicates that the achievement of students' algebraic thinking skills using the Proof-Based Guided Inquiry strategy was maximized. The results of the independent t-test analysis of the post-test results also showed a significant difference in the achievement of algebraic thinking skills between the two groups in Linear Algebra learning, confirming that the Proof-Based Guided Inquiry strategy has a positive effect on the achievement of students' algebraic thinking skills. As shown by the results of research conducted by Darmawati et al. (2025:144), the application of the Guide Inquiry learning model is quite effective in improving students' algebraic thinking skills and learning independence compared to conventional learning models.

Thus, the Proof-Based Guided Inquiry strategy also has an influence on students' critical thinking skills. As shown in the results of research conducted by Susilawati et al. (2024) , who analyzed a systematic literature review of publications between 2019 and 2024 on the application of Guided Inquiry in improving students' thinking skills, concluded that the results of the article analysis showed that the use of the Guided Inquiry learning model significantly improved students' critical thinking skills in understanding scientific concepts. This can be seen from the improvement in students' ability to analyze, evaluate, and interpret information, as well as their ability to construct arguments or opinions based on existing evidence.

Based on the analysis of student responses, it was found that students showed a positive response of 84.8%. This percentage indicates that students generally responded well to learning strategies that emphasize the process of searching, reasoning, and proving mathematical concepts. This shows that students feel that the Guided Inquiry strategy provides a more in-depth learning experience because they not only receive information but are also invited to explore logical steps to arrive at the correct conclusion. This is in line with the results of research conducted by Saufi (2016), which shows that the Guided Inquiry method is more effective in improving student achievement and learning motivation than conventional methods.

However, there were still 15.2% of negative responses from students towards the proof-based Guided Inquiry strategy. This shows that some students feel uncomfortable or experience difficulties when following a learning process that requires active involvement in discovering concepts and performing mathematical proofs. Possible causes include students not being familiar with inquiry-based learning strategies or finding it difficult to construct logical arguments systematically. Therefore, a more in-depth evaluation of the implementation of this strategy is needed to identify the obstacles faced by students, adjust the level of difficulty of the proof to their initial abilities, and design more structured assistance so that the proof-based Guided Inquiry strategy can run more optimally.

CONCLUSION

This study shows that the implementation of the proof-based Guided Inquiry strategy in Linear Algebra learning was carried out very well with an average of 92%. There was a difference in the improvement of algebraic thinking skills between the experimental class and the control class, where the experimental class obtained a moderate average N-Gain (0.401), while the control class obtained a low average N-Gain (0.246). In addition, the achievement of students' algebraic thinking skills in the experimental class was higher than in the control class. Student responses toward the proof-based Guided Inquiry strategy were also positive, with an average of 84.8%. Thus, this strategy was effective in improving learning implementation, algebraic thinking skills, and student engagement.

Future research is recommended to apply the proof-based Guided Inquiry strategy in different mathematical topics or levels of education, involve a larger sample size, and explore additional qualitative aspects such as students' reasoning processes or challenges in constructing proofs. Further studies may also compare this strategy with other inquiry-based or proof-oriented models to provide deeper pedagogical insights.

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