

ANALYSIS OF STUDENTS' DIFFICULTIES ON UNDERSTANDING INTEGRAL CONCEPTS

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ABSTRACT

Integral concepts are among the most challenging topics in mathematics learning because they require students to understand symbolic forms, graphical representations, and the relationship between integrals and derivatives. Many students still experience persistent difficulties due to weak conceptual foundations and limited representational skills. This study aims to analyze the types and causes of students' difficulties in understanding integral concepts in high school mathematics. A qualitative descriptive method was used, supported by diagnostic tests, semi-structured interviews, and classroom observations. The diagnostic test consisted of three questions designed to measure students' conceptual, procedural, representational, technical, and affective difficulties. Interviews were conducted with five students selected based on varying levels of difficulty to explore their thought processes, while observations were carried out to validate the behavioral patterns that emerged during the test and interview sessions. Data were analyzed using the Miles and Huberman model, consisting of data reduction, data display, and conclusion drawing. The results show that conceptual difficulties are the most dominant, particularly in understanding integrals as areas, accumulation processes, and their relationship to derivatives. These difficulties contribute to procedural and technical errors, including incorrect steps, algebraic mistakes, and inaccurate limit substitutions. Representational and affective difficulties, such as anxiety and low confidence, also negatively impact students' performance. The study concludes that students' integral difficulties are multidimensional and interconnected, suggesting the need for instructional approaches that emphasize conceptual reinforcement, multiple representations, and supportive learning environments.

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INTRODUCTION

Mathematics learning plays a crucial role in developing students' logical, analytical, and systematic thinking skills (Wang et al., 2025). One topic of high complexity is integrals, including both definite and indefinite integrals. Integral concepts are not only related to

mathematical operations but also require a deep understanding of derivatives, functions, and limits (Ramawati et al., 2024). However, in reality, many students struggle to grasp integral concepts due to their abstract nature and strong symbolic representation skills (Hoban, 2019).

These difficulties often arise in the form of misconceptions about the geometric meaning of integrals, an inability to connect derivatives and integrals, and errors in applying integral rules to various types of functions (Bašić & Milin, 2022). Furthermore, teaching methods that tend to be procedural and do not emphasize conceptual understanding cause students to focus only on the steps to solve problems without understanding the underlying mathematical meaning (Hussein & Csikos, 2023). As a result, learning outcomes in integrals tend to be low, and conceptual understanding becomes shallow (Renaya Dwi Septiani & Harisman, 2025).

Several previous studies have examined the challenges students face in understanding integrals. Zahroh et al. (2022) demonstrated that 30% of students committed principal errors and 23% made operational errors due to a lack of understanding of basic integral concepts and a weak grasp of prerequisite materials, such as derivatives and algebra. Furthermore, inattention and minimal practice exacerbated learning difficulties. These findings confirm that the primary issue lies in the conceptual understanding of integrals, rather than in the procedural aspects of integration. Furthermore, Fahrurrozi et al. (2022) revealed that 31.03% of students had difficulty understanding basic integral concepts, 37.93% had difficulty using principles, and 48.28% had difficulty solving word problems. The primary causes were misconceptions about the relationship between integrals and derivatives, as well as an inability to apply formulas in various contexts.

On the other hand, Mahayukti et al. (2022) found that in online learning, 60% of students made conceptual errors and 70% made application errors. These errors were primarily caused by confusion in selecting appropriate integration techniques and a poor conceptual understanding, resulting from limited interaction during the learning process.

Similar findings were also presented by Ijuddin et al. (2022) in the *Journal of Numeracy*, which stated that 40 to 70% of students experience misconceptions regarding various integral concepts, particularly regarding the relationship between derivatives and integrals, the fundamental theorem of calculus, and the geometric meaning of integrals as the area under a curve. This study confirms that conceptual misconceptions are the main root of difficulties in understanding integrals, not merely procedural errors.

Although various previous studies have provided insights into the types of errors and misconceptions experienced by learners, most have focused on identifying procedural errors without deeply examining the structure of difficulties in integral conceptual understanding. Furthermore, many studies have been conducted in the context of higher education or online learning, resulting in limited research exploring how students construct integral conceptual understanding in real classroom situations (Rosidah et al., 2024). Therefore, a research gap exists that needs to be addressed through a more in-depth analysis of integral conceptual difficulties, particularly by examining the interrelationships between the symbolic, graphical, and verbal aspects of mathematical representation. In this context, understanding the intrinsic connection between integrals and derivatives becomes essential, as both are fundamentally linked through the Fundamental Theorem of Calculus, which states that an integral is the inverse of a derivative (Mulyani & Siregar, 2025). This means that understanding integrals is inseparable from the concepts of derivatives, limits, and changes in variable values (Sakdiah & Siregar, 2025). While derivatives describe rates of change, integrals represent the accumulation of those changes, such as the area under a curve (Siregar & Siregar, 2025). Since integration rules rely on reconstructing a function prior to differentiation, mastery of derivatives becomes a crucial prerequisite, and failure to connect these concepts often results in misconceptions in

both indefinite and definite integrals (Astuti et al., 2025). Thus, the relationship between integrals and derivatives represents a complementary conceptual foundation that must be emphasized in calculus learning.

This research is highly relevant because understanding the concept of integrals is fundamental to mastering advanced topics in calculus and other applied sciences. Analyzing difficulties in understanding integrals will provide more comprehensive information regarding the root causes of problems in mathematics learning in calculus. The results are expected to serve as a basis for developing learning strategies that emphasize students' understanding of mathematical meaning, connections between concepts, and multiple representations. Therefore, this research not only provides theoretical contributions to the study of mathematics learning difficulties but also has practical value in helping teachers design learning that facilitates conceptual and sustainable understanding of integrals.

Based on the results presented above, this study aims to analyze students' difficulties in understanding integral concepts in mathematics learning. Specifically, this study seeks to identify the types of problems experienced by students, the factors that cause them, and provide an overview of how learning strategies can be adapted to address these difficulties.

METHOD

This study employs a qualitative descriptive approach aimed at identifying and analyzing the forms and causes of students' difficulties in understanding the concepts of definite and indefinite integrals (Kasali et al., 2023). This approach was selected because it aligns with the research objectives, namely to explore in depth students' thought processes when solving integral problems and to uncover the factors that contribute to the emergence of conceptual, procedural, representational, technical, and affective difficulties in learning integral calculus (Rajiun & Nida, 2025).

The research was conducted at a high school in Mandailing Natal Regency that has implemented Integral material in grade 11. School selection was conducted purposively based on teachers' willingness to collaborate and representative student characteristics (Aziz, I. R., Zaura, B., & Umam, 2025). The study participants consisted of 20 students, but the primary data source in the qualitative approach was obtained from diagnostic tests and in depth interviews with five students selected based on their high and low levels of difficulty.

The research instruments consisted of an integral diagnostic test and a semi-structured interview guide. The diagnostic test contained three integral questions designed to measure five indicators of student difficulties, as shown in the following table 1.

Table 1. Indicators of Student Difficulty in Solving Integral Problems

No	Difficulty Category	Indicator
1	Conceptual	<ul style="list-style-type: none"> • Understand the definition of a definite integral. • Understand the meaning of an indefinite integral. • Understand the relationship between an integral and area.
2	Procedural	<ul style="list-style-type: none"> • Correctly apply the steps for solving integrals. • Correctly use the basic rules of integrals.

3	Representational	<ul style="list-style-type: none"> • Apply substitution or partial procedures as needed. • Converting graphs into symbolic or verbal form in an integral context. • Interpreting integral symbols or graphs. • Connecting various representations (graph symbol–verbal).
4	Technical	<ul style="list-style-type: none"> • Perform algebraic operations accurately. • Manipulate symbols correctly.
5	Affective	<ul style="list-style-type: none"> • Demonstrate accuracy in calculations. • Demonstrate a level of anxiety when solving integral problems. • Have or lack confidence in solving integrals. • Demonstrate a positive or negative attitude toward integral problems.

Meanwhile, the interview guidelines were used to further explore the causes of the difficulties identified in the diagnostic test, as well as to strengthen the findings through observations of how students explained the steps they used to solve the problems. Observations were conducted during the interview process to record students' expressions, pauses in thought, hesitations, and spontaneous strategies that emerged when they were asked to explain their answers (Sugiyono, 2019). The interview indicators included: (1) understanding of basic integral concepts; (2) reasons for choosing certain steps or strategies; (3) obstacles in applying integral procedures such as substitution and basic rules; (4) difficulties in interpreting graphs and mathematical symbols; and (5) affective factors, including fear, confusion, or lack of confidence when solving problems.

The research consisted of three stages: data collection, data analysis, and conclusion drawing. In the data collection stage, two main techniques were used, namely tests and interviews (Assya et al., 2025). The instruments employed included descriptive test questions and interview guidelines. The data collection process was carried out in two sequential steps, and the test questions consisted of three modified items, as shown in Table 2.

Table 2. Test questions

No	Question Items
1	Calculate the indefinite integral of $\int(4x^3 - 2x) dx$
2	Calculate the following definite integral of $\int_{-10}^{10} 2x^2 + x dx$
3	Calculate the following definite integral and show its relationship to the area: of $\int_{-2}^2 x^2 + x dx$

The data analysis technique used in this study followed the Miles and Huberman model, which consists of data reduction, data presentation, and conclusion drawing (Sudaryono, 2021). In the data reduction stage, errors in students' test answers were identified, their difficulties were categorized into conceptual, procedural, representational, technical, and affective types, and relevant interview excerpts were transcribed and selected. The final stage involved drawing conclusions to answer the research questions regarding the types of difficulties experienced by the students (Dharshinni & Saleh, 2021).

RESULTS AND DISCUSSION

Results

To clarify the patterns of difficulties experienced by students on each integral diagnostic test item, the error types were grouped according to five difficulty indicators: conceptual, procedural, representational, technical, and affective errors. The following table provides a summary of the errors displayed by the five students (S1–S5) when completing the three integral problems, allowing the differences in the characteristics of the difficulties experienced by each student to be observed.

Table 3. Grouping of types of difficulties

No	Student Code	Question 1	Question 2	Question 3
1	S1	TD	PD	CD
2	S2	CD	TD	RD
3	S3	PD	KD	KD
4	S4	TD	KD	TD
5	S5	AD	AD	AD

Information:

- CD : Conceptual Difficulties
- PD : Procedural Difficulties
- RD : Representational Difficulties
- TD : Technical Difficulties
- AD : Affective Difficulties

Based on the analysis presented in the Error Type Grouping Table, each student demonstrated a different pattern of difficulty for each type of problem. S1 exhibited technical, procedural, and conceptual errors, which aligned with interview results indicating limited understanding of the meaning of integrals and frequent calculation mistakes. S2 encountered conceptual, technical, and representational errors, particularly in connecting symbolic forms with their graphical representations. S3 showed a predominance of procedural and conceptual errors due to confusion in determining appropriate solution steps and a weak understanding of the relationship between integrals, function properties, and area.

Meanwhile, S4 exhibited technical and procedural errors that were influenced by carelessness and a tendency to omit important steps due to working hastily. In contrast, S5 demonstrated a unique pattern dominated by affective errors, such as doubt, anxiety, and lack of confidence, which resulted in incomplete or unfinished answers. Overall, these findings indicate that students' difficulties in understanding integrals are influenced by a combination of conceptual, procedural, technical, representational, and affective factors.

Based on the analysis, conceptual difficulties were the most dominant type of difficulty at 33.33%, followed by technical difficulties at 26.67%. This indicates that obstacles in conceptual understanding occurred slightly more frequently than technical errors in the integral-solving process. Furthermore, affective difficulties were recorded at 20.00%, suggesting that psychological factors such as anxiety and lack of confidence also contributed to students' low performance.

Procedural difficulties occurred at 13.33%, indicating that some students still struggled to determine the correct steps for solving the problems. Representational difficulties were the least common, occurring at 6.67%, suggesting that only a small number of students experienced challenges in connecting graphical, symbolic, and verbal representations. All of these difficulties will be explained further in the following section.

Conceptual difficulties were seen in subjects S1 and S2, who showed an inability to understand the meaning of indefinite integrals and the relationship between integrals and the concept of antiderivative functions presented in Figures 1 and 2.

Handwritten work for Figure 1:

$$1) \int 4x^3 - 2x \, dx = 4x^4 + 2x$$

$$= 3x^4 + 2x$$

$$= 3x^2 + 2$$

Figure 1. Student 2 Answers that Describe Conceptual Errors in Integral Material

Handwritten work for Figure 2:

$$3) \int_{-1}^2 x^2 + x \, dx = \int x^2 \, dx + \int x \, dx$$

$$= 2x + x$$

$$= 3x$$

Figure 2. Student 1 Answers that Describe Conceptual Errors in Integral Material

Conceptual difficulties are evident in the students’ answers, which treat integrals merely as a process of transforming algebraic expressions without understanding the fundamental concept of integrals as areas or proper antiderivatives. In Question 3, the students failed to apply the upper and lower limits and did not construct the resulting antiderivative correctly, indicating that they did not understand that definite integrals must yield numerical values rather than functional forms. Furthermore, the students did not recognize that indefinite integrals require the addition of a constant (+C) and were unable to connect integrals to their geometric interpretation. These errors confirm that the students did not fully understand the concept of integrals and relied solely on algebraic manipulation without meaningful conceptual grounding.

In addition to conceptual difficulties, some students also experienced obstacles in procedural, representational, technical, and affective aspects. In the procedural aspect, S3 appeared to make mistakes in determining the solution steps, for example, choosing the wrong method or inconsistently following the integration sequence, resulting in the answer not reaching the final stage and errors in writing the problem as presented in Figure 3 below.

Handwritten work for Figure 3:

$$1) \int (4x - 2x) \, dx = \int 4x \, dx - \int 2x \, dx$$

$$= x - 2$$

Figure 3. Student Answers Showing Procedural Difficulties in Indefinite Integral Problems

The answers in the image indicate that the student experienced procedural difficulties, specifically an inability to follow the proper steps for solving integrals in accordance with the rules. The student separated the integrand into two parts but did not correctly apply the basic rules of integration. For example, $\int 4x \, dx$ should result in $2x^2$ and $\int 2x \, dx$ should result in x^2 , but the student wrote them directly as $x - 2$ without performing the correct integration process. This error shows that the student did not understand the step-by-step procedure for solving integrals, including increasing the exponent, dividing by the new exponent, and writing the constant. Thus, this mistake is a clear example of procedural difficulty, in which the student is unable to correctly apply the algorithms and rules of integral operations.

Meanwhile, representational difficulties were observed in S2 on question number 3, who experienced obstacles in connecting symbolic forms, graphs, and area representations, including difficulty visualizing the area under the curve and determining the integral limits accurately as presented in Figure 4 below.

$$\begin{aligned}
 3. \int_{-2}^2 (u^2 + 2u) \, du &= \int_{-2}^2 u^2 \, du - \int_{-2}^2 f(u) \\
 &= \int_{-2}^2 u^2 \, du + \int_{-2}^2 2u \, du \\
 &= \frac{1}{3} u^3 + \frac{1}{2} 2u^2 \\
 \\
 f(2) &= \frac{1}{3} (2)^3 + \frac{1}{2} (2)^2 \\
 &= \frac{8}{3} + \frac{4}{2} \\
 &= \frac{8}{3} + 2 \\
 &= \frac{14}{3} \\
 \\
 f(-2) &= \frac{1}{3} (-2)^3 + \frac{1}{2} (-2)^2 \\
 &= -\frac{8}{3} + \frac{1}{2} (4) \\
 &= -\frac{8}{3} + 2 \\
 &= -\frac{2}{3} \\
 \\
 \text{Sehingga :} \\
 \int_{-2}^2 u^2 + 2u \, dx &= f(2) - f(-2) \\
 &= \frac{14}{3} - \left(-\frac{2}{3}\right) \\
 &= \frac{14}{3} + \frac{2}{3} \\
 &= \frac{16}{3}
 \end{aligned}$$

Figure 4. Student Answers Showing Representational Difficulties in Solving Definite Integrals

Based on the answers shown in the figure, the student experiences representational difficulties, namely difficulties in converting integral expressions into correct mathematical representations and translating procedures into appropriate symbolic forms. The student attempts to separate the integral into two parts and calculate the values of $f(2)$ and $f(-2)$, but there are inconsistencies in symbol writing, placement of limits, and the use of integral notation. For example, the student repeatedly rewrites the integrals in varying forms, confuses the symbols u and x , and includes limits that do not always correspond to the previous integral expression. In addition, the student appears confused when writing substitution results and procedural steps, making the mathematical representation incoherent and difficult to understand. These errors indicate that the student is not yet able to use symbols, function forms, and integral notation accurately, thus falling into the category of representational difficulties.

In the technical aspect, students S1 and S4 showed calculation errors, misplaced signs, and mistakes in algebraic manipulation and limit substitution, which resulted in inaccurate numerical answers. Furthermore, affective difficulties were also a significant factor,

particularly for S5, who showed anxiety, doubt, and lack of confidence when working on the problems, leading to incomplete answers that did not reflect their actual abilities. These five types of difficulties are interconnected and demonstrate that understanding integrals requires not only mastery of concepts and procedures, but also emotional readiness and adequate representational and technical skills.

To complement the quantitative findings, this study also collected qualitative data through semi-structured interviews and direct observations of the learning process. These qualitative data were used to more comprehensively identify the sources of students' difficulties and to provide contextual explanations of the behaviors, strategies, and obstacles that emerged during their learning of integral concepts. The results of the qualitative analysis are presented in the following section.

To gain a more comprehensive understanding of the forms and factors contributing to students' difficulties in learning integrals, semi-structured interviews were conducted with five respondents. These interviews aimed to directly explore students' learning experiences, particularly regarding conceptual, procedural, representational, technical, and psychological aspects. A summary of the qualitative findings, along with representative student quotes, is presented in Table 4.

Table 4. Results of Semi structured Interviews on Integral Learning Difficulties

Difficulty Aspect	Thematic Interview Results	Representative Quotes from Students
Conceptual Difficulties	Most students do not fully understand the meaning of integrals. They consider integrals merely the reverse process of differentiation without understanding their geometric meaning, which is the area under a curve.	"I know that an integral is the inverse of a derivative, but I don't understand what it means. If I were to ask why the result is like that, I wouldn't be able to explain it." (Student 1)
Procedural Difficulties	Students often struggle to determine the steps to solve a problem, particularly when selecting the correct method, such as substitution or partial substitution. Mistakes frequently occur due to a lack of understanding of the method's requirements.	"I'm often confused about when to use substitution or just integrate. Sometimes my steps are wrong from the start." (Student 2)
Representational Difficulties	Students struggle to connect algebraic expressions to function graphs and interpret integral results. They also struggle to visualize the relationship between symbols and conceptual meaning.	"When it comes to using graphs, I find it difficult to imagine what the area or area under the curve is like." (Student 3)
Technical Difficulties /Counting	Technical errors appeared in the calculations, particularly in algebraic operations and limit substitutions. A lack of	"I often mis-sign or forget to substitute the upper and lower limits, even though I already know the formula." (Student 4)

	precision and basic algebraic knowledge exacerbated the errors.	
Psychological/ Affective Difficulties	Some students feel anxious and lack confidence when working on integral problems. They find the topic difficult and tend to give up early.	"When I see a long integral problem, I immediately get scared. It feels really hard." (Student 5)

Table 4 shows that, based on the interview results, conceptual and procedural difficulties are the primary obstacles students face in understanding integration material. Students tend to memorize formulas without grasping their mathematical meaning. Furthermore, representational difficulties exacerbate their challenges, as students are unable to connect symbols with their visual or geometric interpretations. Technical errors frequently occur due to inaccuracy and weak foundational algebra skills, while affective factors such as anxiety and low self-confidence also contribute to poor learning outcomes.

These findings reinforce the results of the quantitative analysis, indicating that weak conceptual understanding directly contributes to procedural and technical errors, and they also demonstrate that psychological factors play a significant role in successful integral learning. The connection between the interview findings and the quantitative data became even clearer when confirmed through classroom observations. The observations revealed that most students experienced difficulties in learning integrals across the five examined aspects. Conceptually, students had not yet understood integrals as representations of the area under a curve and continued to rely on memorizing formulas. This was evident when only a few students were able to explain the geometric meaning of integrals when prompted with guiding questions. Procedurally, students appeared unsure in determining the appropriate solution steps, particularly when substitution methods were required, leading them to wait for examples from the teacher.

Students experienced representational difficulties when connecting function graphs with symbolic forms of integrals, including determining integral limits and visualizing the regions under the curve. Technically, errors were still found in basic algebraic operations and in substituting the upper and lower limits, resulting in inaccurate answers. Psychologically, some students appeared to lack confidence, were passive, and showed anxiety when completing the exercises. To ensure the validity of the findings, triangulation was conducted by comparing the observation and interview data to examine the consistency of the results, thereby making the interpretation of students' difficulties in learning integrals more comprehensive and credible. A summary of the triangulated findings across the five aspects of difficulty is presented in Table 5.

Table 5. Results of Semi structured Interviews on Integral Learning Difficulties

Difficulty Aspect	Interview Findings	Observation Findings	Triangulation Results
Conceptual Difficulties	Students often do not understand the meaning of integrals as representing area; instead, they memorize the formula without grasping the	Students are unable to explain the concept when the teacher asks, and are confused about determining the meaning of the integral symbol.	Findings from both data sources show a consistent pattern, where conceptual difficulties appear to emerge predominantly.

	relationship between integrals and derivatives.		
Procedural Difficulties	Students are confused about choosing a method (substitution/partial) and make mistakes in the systematic steps of the solution.	Students are hesitant to follow the steps of the solution; they often wait for examples from the teacher.	Findings from interviews and observations indicate alignment, with procedural difficulties emerging predominantly.
Representational Difficulties	Students struggle to connect graphs with symbols, often due to difficulty visualizing the area under the curve.	Students incorrectly determine the limits on the graph and are unable to identify the integral region.	Consistently, representational difficulties arise in both techniques.
Technical Difficulties /Counting	Students often make substitution errors, sign errors, and errors in algebraic operations.	Many calculation errors occur, especially in substitutions involving upper and lower limits and power manipulation.	Consistent, Technical errors stem from weak algebraic foundations.
Psychological/Affective Difficulties	Students feel anxious and afraid when facing long and complicated integral problems.	Students appear passive, lack confidence, and are reluctant to do the exercises.	Consistent, Affective factors influence how students learn integrally.

Based on the triangulation results, the interview and observation findings show a consistent pattern across all aspects of difficulty. This alignment confirms that students' primary challenges lie in the conceptual and procedural domains, which subsequently affect their representational, technical, and affective abilities. Thus, the triangulation provides a strong basis for validating the interpretation of the overall research findings.

Discussions

The results of the study indicate that students' difficulties in understanding integrals predominantly arise in the conceptual, procedural, representational, technical, and psychological aspects. In the conceptual domain, students were unable to connect integrals with the notion of area, the process of accumulation, or their relationship to derivatives. This finding is consistent with Fahrurrozi et al. (Fahrurrozi et al., 2022), who reported that weak conceptual understanding is a major factor contributing to students' difficulties with integrals at the high school level.

These conceptual difficulties directly contribute to the emergence of procedural and technical errors. Interviews and observations revealed that students frequently miscalculated integral limits, made inappropriate substitutions, and struggled with algebraic manipulation (Wibawa & Winarsih, 2020). These findings align with Jabnabillah (2022), who asserted that inadequate conceptual understanding of integrals affects students' solution procedures, indicating that many technical errors are, in fact, consequences of underlying conceptual weaknesses.

In the representational aspect, students still have difficulty connecting function graphs with integrals, particularly when interpreting integrals as limits of Riemann sums. This is consistent with Hoban's (Hoban, 2019) analysis, which shows that many students become confined to formal symbols without understanding the structure of integrals as a visual and intuitive process of accumulation. Such limited representational ability weakens students' capacity to move between different forms of representation (graphical, symbolic, and verbal), which is essential in modern calculus learning.

From a psychological perspective, math anxiety has been shown to be a significant inhibiting factor. Anxious students tend to avoid complex integral problems, hesitate to begin solving them, and often wait for examples from the teacher. This finding is consistent with research by Rahman et al (2025), which showed that math anxiety negatively affects motivation, self-efficacy, and higher-order problem-solving abilities. Thus, the affective aspect strengthens the cognitive barriers experienced by students.

Quantitative and qualitative triangulation indicates that difficulties in learning integrals are multidimensional, with conceptual weaknesses leading to procedural errors, representational difficulties hindering the interpretation of graphs and symbols, and anxiety affecting students' decision-making. These findings highlight the need for instructional approaches that emphasize conceptual reinforcement, the use of visual representations, and affective support. However, this study has several limitations, including a small sample size drawn from a single school, potential bias in interviews and observations, and a cross-sectional design that does not capture long-term development. Future studies could expand the research context, employ longitudinal designs, and examine the effectiveness of concept- and representation-based instructional approaches in addressing students' difficulties with integrals.

CONCLUSION

Based on the results of diagnostic tests, interviews, and observations, this study indicates that students' difficulties in understanding integrals are multidimensional, encompassing conceptual, procedural, representational, technical, and affective aspects. Conceptual difficulties are the most dominant, particularly in understanding integrals as areas, accumulation processes, and their relationship to derivatives, which subsequently trigger procedural and technical errors in determining solution steps, applying integral rules, and performing algebraic manipulations and limit substitutions. Representational difficulties are also evident when students are unable to connect symbolic forms with graphical representations, while affective factors such as anxiety and lack of self-confidence further hinder their ability to solve problems. Based on these findings, integral learning should emphasize conceptual reinforcement through multiple representations and visualizations, structured exercises to minimize technical errors, and affective support to strengthen students' confidence. Further research is recommended to involve larger samples, employ longitudinal designs, and examine the effectiveness of representation-based instructional models and anxiety-reduction interventions to obtain more comprehensive and applicable findings.

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