

ANALYSIS OF STUDENTS' MATHEMATICAL REASONING ABILITY ON JUNIOR HIGH SCHOOL

Ulfiani Usman¹, Dwi Juniati², Siti Khabibah³

¹Universitas Negeri Surabaya, Jl. Ketintang Wiyata, Surabaya, Indonesia.

ulfiani.usman32@gmail.com

²Universitas Negeri Surabaya, Jl. Ketintang Wiyata, Surabaya, Indonesia.

dwijuniati@unesa.ac.id

³Universitas Negeri Surabaya, Jl. Ketintang Wiyata, Surabaya, Indonesia.

sitikhabibah@unesa.ac.id

ARTICLE INFO

Article history:

Received Dec 15, 2025

Revised Dec 16, 2025

Accepted Dec 18, 2025

Keywords:

Mathematical Reasoning
Ability
Pythagorean Theorem
Junior High School

ABSTRACT

Students' mathematical reasoning ability (MRA) is an essential component of mathematics learning, particularly for solving non-routine problems. However, many junior high school students still rely on procedural strategies without sufficient conceptual understanding, resulting in weak reasoning skills, especially in geometry topics such as the Pythagorean Theorem, indicating the need for a detailed analysis of students' mathematical reasoning ability. This study aims to analyze junior high school students' mathematical reasoning ability on the Pythagorean Theorem based on four indicators: proposing conjectures, performing mathematical transformations, providing logical justification, and drawing conclusions. A descriptive qualitative approach was employed involving 31 eighth-grade students of SMP Negeri 1 Raha selected through purposive sampling. Data were collected using an open-ended mathematical reasoning ability test related to the Pythagorean Theorem and supported by semi-structured interviews. Data analysis was conducted using Miles and Huberman's interactive model, consisting of data reduction, data display, and conclusion drawing and verification. The results indicate that 6% of students demonstrated high mathematical reasoning ability, 8% were categorized as moderate, and 86% were classified as low. Most students experienced difficulties in formulating conjectures, transforming contextual problems into mathematical models, providing logical justification, and drawing valid conclusions. In conclusion, students' mathematical reasoning ability on the Pythagorean Theorem remains relatively low. Therefore, instructional strategies that emphasize conceptual understanding, reasoning processes, justification, and reflective thinking are necessary to improve students' mathematical reasoning abilities.

Copyright © 2026 IKIP Siliwangi.

All rights reserved.

Corresponding Author:

Ulfiani Usman,
Department of Mathematics Education,
Universitas Negeri Surabaya,
Jl. Ketintang Wiyata, Surabaya, Indonesia.
Email: ulfiani.usman32@gmail.com

How to Cite:

Usman, U., Juniati, D., & Khabibah, S. (2026). Analysis of Students' Mathematical Reasoning Ability on Junior High School. *JIML*, 9(1), 122-132.

INTRODUCTION

Mathematical reasoning ability is a fundamental component of mathematics learning, enabling students to think logically, analyze relationships, and justify mathematical conclusions. Through reasoning, students do not merely apply formulas procedurally but also develop an understanding of why mathematical concepts work and how they can be applied in various

contexts. According to the National Council of Teachers of Mathematics (NCTM, 2000), mathematical reasoning is one of the essential process standards that supports students in constructing knowledge and solving problems meaningfully.

Despite its importance, many students experience difficulties in developing reasoning skills. In classroom practice, students often rely on memorization and imitation of examples provided by teachers, without fully understanding underlying concepts. Consequently, they struggle to connect information, formulate arguments, and draw logical conclusions when faced with non-routine problems. This condition indicates that students' mathematical reasoning ability remains relatively low, particularly in topics that require conceptual understanding and logical thinking.

The Pythagorean Theorem is a topic that demands strong reasoning skills. Mastery of this topic involves understanding the relationships among the sides of right triangles, performing appropriate mathematical transformations, and providing logical justifications for solutions. Instead of merely substituting numbers into formulas, students are expected to analyze problem conditions, determine strategies, and explain their reasoning. However, previous studies have reported common reasoning errors, such as misidentifying triangle elements, applying formulas incorrectly, or failing to justify answers logically (Polya, 1945; NCTM, 2000).

International assessments highlight the gap in students' reasoning abilities. According to the 2022 PISA results, Indonesian students' mathematics and problem-solving performance remain below the OECD average:

Table 1. PISA 2023 Results (Volume I)

Assessment	Indonesia Score	OECD Average Score
Mathematics Literacy	366	472
Reading Literacy	359	476
Science Literacy	383	485
Creative Thinking (\geq Level 3)	31%	78%

OECD (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*.

These results reflect challenges in applying mathematical concepts beyond routine exercises. National studies also report that students often struggle with constructing arguments, performing mathematical transformations, and drawing valid conclusions (Mullis, Martin, Foy, & Hooper, 2020).

Recent research focusing on geometry and the Pythagorean Theorem provides further evidence of students' reasoning difficulties. Khainingsih, Setiawan, and Rahayu (2025) found varied student performance in constructing logical arguments and justifications when solving open-ended Pythagorean problems. Similarly, Ramdan and Roesdiana (2025) revealed low competency in key reasoning indicators, including generalization, mathematical manipulation, and drawing valid conclusions. Studies investigating instructional approaches, such as scientific approaches and Contextual Teaching and Learning, suggest that these methods can significantly improve students' reasoning abilities compared to conventional teaching (Santoso, et al, 2023).

These findings indicate that, despite instructional efforts, students still encounter difficulties in reasoning processes particularly in proposing conjectures, performing mathematical transformations, providing logical justification, and drawing valid conclusions when solving Pythagorean problems. Preliminary observations in junior high schools show that these difficulties are often linked to teacher-centered instruction, which limits opportunities for

students to explore ideas, argue logically, and reflect on solutions. As a result, students often perceive mathematics as a collection of formulas rather than a coherent logical system.

Given these conditions, a detailed analysis of students' mathematical reasoning ability is necessary to identify their strengths and weaknesses. Such analysis can guide teachers in designing instructional strategies that emphasize reasoning, conceptual understanding, justification, and conclusion drawing. Therefore, this study aims to analyze the mathematical reasoning ability of junior high school students on the Pythagorean Theorem, focusing on the four reasoning indicators: proposing conjectures, performing mathematical transformations, providing logical justification, and drawing conclusions. The findings are expected to provide insights into students' reasoning profiles and inform the development of teaching strategies that enhance mathematical reasoning skills.

METHOD

This study employs a descriptive qualitative research method to analyze students' mathematical reasoning ability on the topic of the Pythagorean Theorem. A qualitative approach is chosen to explore in depth how students reason, justify solutions, perform mathematical transformations, and draw conclusions based on clearly defined reasoning indicators.

The population of this study consists of junior high school students (Grade VIII) at SMP Negeri 1 Raha. Participants were selected using purposive sampling, resulting in a total of 31 students taking part in the study. To gain a comprehensive understanding of students' reasoning abilities, three students were selected one from each category of high, medium, and low reasoning ability to represent the responses in data analysis. This classification was determined based on the four indicators of mathematical reasoning:

1. Proposing conjectures
2. Performing mathematical transformations
3. Providing logical justification
4. Drawing conclusions

The instruments used in this study include:

1. **Mathematical Reasoning Test:** A set of subjective problem-solving questions related to the Pythagorean Theorem, designed to assess students' abilities according to the four reasoning indicators mentioned above. The test focuses on:
 - a. Identifying known elements, questions, and sufficiency of information.
 - b. Formulating strategies or constructing mathematical approaches to solve problems.
 - c. Applying mathematical transformations appropriately.
 - d. Providing logical justification and interpreting results accurately.
2. **Interviews:** Semi-structured interviews were conducted to gather qualitative data and clarify students' thought processes, reasoning strategies, and difficulties encountered during problem solving.
3. **Instrument Validation:** The test instrument was developed by the researcher specifically for this study and has undergone validation by expert judgment, as well as testing for content validity, reliability, distinguishing power, and difficulty index to ensure its appropriateness for measuring students' mathematical reasoning ability. The research process was carried out in the following steps:

The research process was carried out in the following steps:

1. Preparation Stage

- a. Designing reasoning test questions and interview guidelines.
- b. Validating research instruments through expert judgment and prior studies.

2. Data Collection

- a. Administering the reasoning test to 31 students.
- b. Selecting three representative students (high, medium, and low reasoning ability) for in-depth interviews.
- c. Conducting interviews to explore reasoning strategies, problem-solving approaches, and difficulties faced by students.

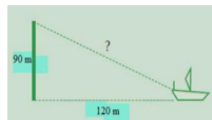
3. Data Analysis

The collected data were analyzed using Miles and Huberman's (1994) interactive model, which consists of three stages: data reduction, data display, and conclusion drawing and verification.

- a. Data reduction was carried out by selecting, focusing, simplifying, and transforming raw data obtained from students' written responses and interview transcripts. At this stage, essential information related to the four indicators of mathematical reasoning ability was identified, while irrelevant data were eliminated. Key reasoning behaviors, errors, and strategies were coded and categorized.
- b. Data display involved organizing the reduced data into tables, figures, and descriptive narratives to identify patterns in students' mathematical reasoning. Students' test scores were summarized statistically to determine the mean, standard deviation, and score distribution. Based on these results, students were classified into high, moderate, and low reasoning categories. Representative student responses from each category were displayed to illustrate their reasoning characteristics.
- c. Conclusion drawing and verification were conducted by interpreting patterns found in the data and validating the findings through triangulation between test results and interview data. This process ensured the credibility of the findings and allowed the researcher to draw conclusions regarding students' mathematical reasoning profiles and the factors influencing their mathematical reasoning ability.

The following figure presents sample questions used to assess students' mathematical reasoning skills on the Pythagorean Theorem.

1. Seorang nakhoda kapal melihat puncak mercusuar yang berjarak 120 meter dari kapal. Jika diketahui tinggi mercusuar 90 meter. Tentukan jarak nakhoda dari puncak mercusuar tersebut! (Tuliskan dugaan Anda untuk menyelesaikan permasalahan ini).



2. Setiap pagi Ajeng berjalan kaki dari rumah menuju sekolah. Dari rumah, Ajeng berjalan sejauh 0,8 km ke arah timur, kemudian dilanjutkan 0,6 km ke arah utara. Tentukan jarak terdekat rumah Ajeng ke sekolah! (Tuliskan dugaan Anda untuk menyelesaikan permasalahan ini).
3. Darwin akan mendirikan tenda untuk berkemah, ukuran penampang depan tenda tersebut berbentuk segitiga sama kaki dengan panjang alasnya 6 m dan kedua sisi lainnya berukuran 5 m. Tentukan ukuran panjang minimal tiang yang akan dibutuhkan Darwin untuk merivangga tenda tersebut! (Tuliskan dugaan Anda untuk menyelesaikan permasalahan ini).
4. Pak Abdi ingin membuat seluncuran permainan anak-anak di taman. Menurut teman Pak Abdi, seluncuran yang bagus membentuk sudut kemiringan antara tanah dengan seluncurannya adalah 30° . Panjang seluncuran yang ingin dibuat Pak Abdi adalah 8 meter. Berapakah tinggi seluncuran tersebut supaya tepat membentuk sudut 30° ? (Tuliskan dugaan Anda untuk menyelesaikan permasalahan ini).

Figure 1. Mathematical Reasoning Ability Question

RESULTS AND DISCUSSION

Results

The results of this study are based on students' mathematical reasoning ability test scores, which were analyzed to obtain the maximum score, minimum score, mean, and standard deviation.

Table 2. Results of the Mathematical Reasoning Ability Test

Number of Students	Maximum Score	Minimum Score	Standard Deviation	Mean
31	63	23	10,30	46,13

The results show that the mean score of students' mathematical reasoning ability was 46.13, with a standard deviation of 10.30. The highest score achieved by students was 63, while the lowest score was 23. These results indicate that students' mathematical reasoning ability tends to vary, with a relatively low average score.

Based on Sudijono's (2011) categorization method, students' mathematical reasoning abilities were classified into three categories: high, medium, and low, as presented in Table 3.

Table 3. Categorization of Students' Mathematical Reasoning Ability

Category	Score Interval	Number of Students	Percentage
High	$X \geq 56,43$	2	6%
Medium	$35,83 \leq X < 56,43$	3	8%
Low	$X < 35,83$	26	86%

These findings indicate that the majority of students are categorized as having low mathematical reasoning ability, which suggests that students experience difficulties in proposing conjectures, performing mathematical transformations, providing logical justification, and drawing valid conclusions. Only 2 students demonstrated high mathematical reasoning ability, 3 students showed moderate ability, while 26 students were classified as low.

Dik: jarak kapal ke mercusuar = 120 m
 Tinggi mercusuar = 90 m
 Dit: jarak nahkoda ke puncak mercusuar?
 peny: soal itu bisa dijawab dengan menggunakan teorema pythagoras karena bentuknya segitiga siku-siku

$$j^2 = 90^2 + 120^2$$

$$j^2 = 8100 + 14400$$

$$j^2 = 225.000$$

$$j = \sqrt{225.000}$$

$$j = 150$$

untuk menjawabku benar: $90^2 + 120^2 = 150^2$
 $8100 + 14400 = 225.000$
 $225.000 = 225.000$ (benar).
 jadi, jarak nahkoda ke puncak mercusuar adalah 150 meter

Figure 2. High Category Students' Answers for Question Number 1

Students classified in this category exhibited high levels of mathematical reasoning ability. They were able to formulate relevant conjectures from the information provided, apply appropriate mathematical transformations, and present solution procedures in a logical and systematic manner. Furthermore, students were capable of justifying their answers and drawing

conclusions that aligned with the given statements. Nevertheless, minor inaccuracies were still identified, particularly due to insufficient carefulness in performing calculations and reviewing the results. As expressed the student, "I applied the correct approach, but I did not recheck my calculations, which led to a small error." This finding indicates that students need to strengthen their habit of verifying answers to ensure accuracy.

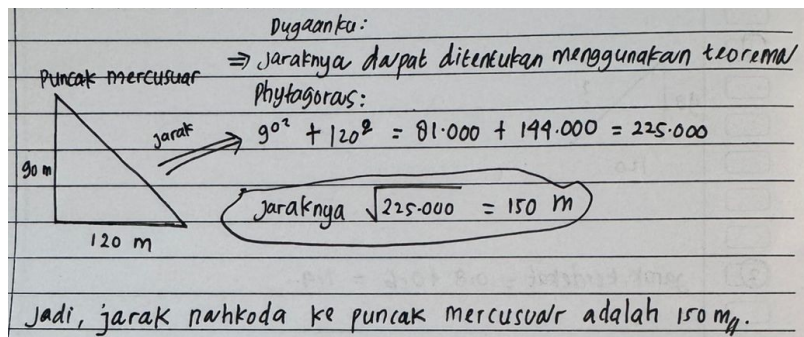


Figure 3. Medium Category Students' Answers for Question Number 1

Students in this category were able to obtain answers to the problems, but their mathematical reasoning was not shown consistently throughout the solution process. They frequently skipped essential steps, hesitated in choosing suitable mathematical transformations, and demonstrated low confidence in their reasoning. As expressed by one student, "I was not sure which formula to use, so I tried several approaches." This suggests that students tended to depend on trial and error methods rather than applying a systematic and logical reasoning process.

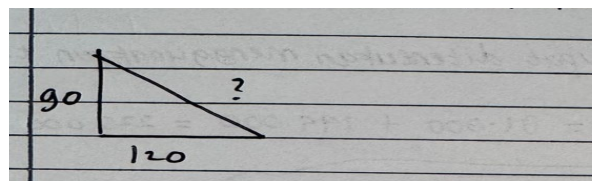


Figure 4. Low Category Students' Answers for Question Number 1

The majority of students (26 out of 31) were placed in this category, indicating substantial challenges in demonstrating mathematical reasoning ability. These students often struggled to understand the problems presented and were unable to formulate suitable conjectures or identify appropriate solution procedures. As expressed by one student, "I don't really know how to start." This finding suggests that students have limited familiarity with reasoning strategies and experience difficulties in systematically analyzing mathematical situations.

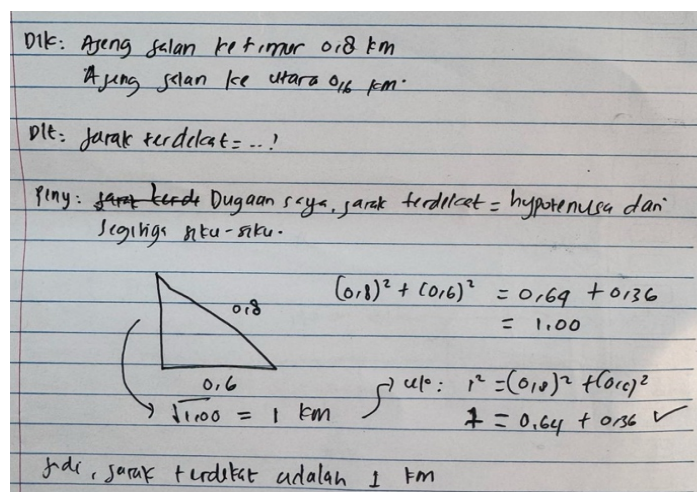


Figure 5. High Category Students' Answers for Question Number 2

For problem number two, the student correctly understood that the shortest distance could be found using the Pythagorean Theorem, but initially drew the triangle incorrectly. When asked about this, the student said, *“I knew the shortest distance was the hypotenuse, but I drew the triangle wrong at first.”* This highlights that the student’s reasoning was strong, as they were able to propose the conjecture, transform the problem into a mathematical model, and draw the correct conclusion, even though a minor visualization error occurred. Justification was simple but still present.

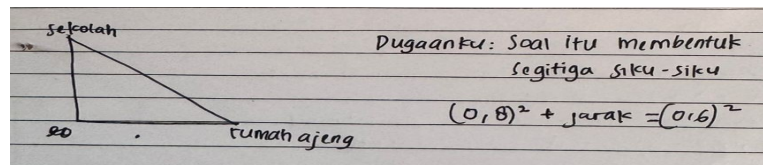


Figure 6. Medium Category Students' Answers for Question Number 2

For the second question, students recognized that the problem involved a triangle, but displayed minimal engagement in applying the Pythagorean Theorem. One student stated, *“I guessed it would be a triangle, but I didn’t know how to calculate the distance.”* This indicates that students could make a basic conjecture but had difficulty transforming the problem into a correct mathematical model, providing justification, and drawing a valid conclusion.

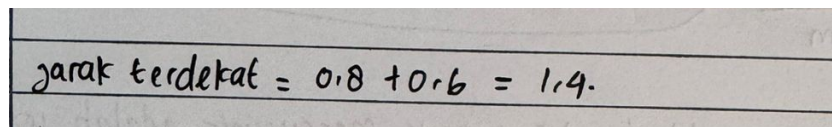


Figure 7. Low Category Students' Answers for Question Number 2

In problem number two, the student did not initially recognize that the situation formed a right triangle. As a result, the student did not apply the Pythagorean Theorem correctly and attempted to determine the distance by simply adding the two given distances. When asked about this mistake, the student stated, *“I thought the shortest distance was just adding the distances because Ajeng walked east and then north.”* This reflects a gap in reasoning, as students were unable to propose a correct conjecture, transform the problem into a mathematical model, provide justification, or draw a valid conclusion.

Based on the analysis, students in the high reasoning category demonstrated the ability to propose appropriate conjectures, perform correct mathematical transformations, justify solution steps logically, and draw conclusions consistent with the problem context. Factors contributing to high MRA include strong conceptual understanding, familiarity with non-routine problems, confidence in reasoning, and the habit of checking solutions. Students in the moderate reasoning category were generally able to solve problems but showed inconsistencies in applying reasoning indicators. They often hesitated in choosing strategies and provided limited justification. Factors influencing moderate MRA include partial conceptual understanding, reliance on trial-and-error strategies, and lack of confidence in explaining reasoning. Students in the low reasoning category experienced significant difficulties across all reasoning indicators. They struggled to understand problem situations, failed to construct appropriate conjectures, and were unable to justify or conclude solutions correctly. Contributing factors include weak conceptual understanding, dependence on memorization, limited exposure to reasoning-based tasks, and teacher centered learning experiences.

Discussions

The findings indicate that students face difficulties in various aspects of mathematical reasoning, particularly across the four main reasoning indicators: proposing conjectures,

performing mathematical transformations, providing logical justification, and drawing conclusions.

1. Proposing Conjectures

Many students, especially those in the low and moderate categories, experienced difficulties in proposing initial conjectures based on the given information. Students were often unable to predict the correct structure of the problem situation, such as identifying that Ajeng's walking path formed a right triangle. In contrast, students in the high category were generally able to propose appropriate conjectures, although minor inaccuracies sometimes occurred in visualizing the situation. These findings are consistent with Nurlinda et al. (2024), who reported that students with low mathematical reasoning ability encountered difficulties in generating initial conjectures when solving geometry problems, while students with higher reasoning ability were more capable of predicting mathematical relationships accurately. Similar results were also reported by Nuraini et al. (2021), who found that students' difficulties in conjecturing were strongly related to weak conceptual understanding of geometric relationships.

2. Performing Mathematical Transformations

Students in the low and moderate categories frequently failed to transform contextual information into appropriate mathematical models. Some students misidentified the sides of the triangle or applied incorrect procedures when using the Pythagorean Theorem. Conversely, high-category students were able to perform mathematical transformations correctly, although minor errors still occurred due to inaccuracies in drawing or calculation. These findings align with Mullis et al. (2020), which indicated that many students struggle to translate real-world situations into correct mathematical representations, particularly in geometry-related contexts. Similarly, OECD (2019) reported that students' limited mathematical modeling skills significantly affect their performance in contextual geometry problems.

3. Providing Logical Justification

Students in the medium and low categories showed limited ability to provide logical justification for their solution steps. Many students provided final answers without explaining the reasoning behind their procedures. High-category students were able to justify their solutions logically, although in some cases the explanations remained brief and lacked depth. This condition is consistent with the findings of Santoso et al. (2023), who reported that students often focus on obtaining final answers rather than articulating the reasoning process in geometry problem solving. Stylianides (2018) also emphasized that insufficient exposure to justification-based tasks results in weak mathematical argumentation skills among students.

4. Drawing Conclusions

The conclusions drawn by students in the medium and low categories were often inconsistent with the information provided in the problem. These students experienced difficulties in linking calculation results to the context of the problem and formulating valid conclusions. High-category students generally drew correct conclusions, although minor errors were still found due to insufficient rechecking of answers. Similar results were reported by Millah and Hidayah (2024), who found that students with high mathematical reasoning ability were able to draw logical conclusions, while students with low reasoning ability frequently failed to connect mathematical results with contextual meaning. In addition, Kurniawati et al. (2022) noted that students' ability to draw valid conclusions is closely related to their reflective thinking habits.

Overall, these findings suggest that students' mathematical reasoning skills still require significant improvement, particularly in proposing conjectures, performing mathematical transformations, providing logical justification, and drawing conclusions. Teachers should emphasize problem-solving strategies that highlight reasoning steps, including visualization, systematic calculations, and reflection on answers, to strengthen students' reasoning abilities. Although students in the high category demonstrated strong reasoning skills, attention is still needed to reduce minor errors and reinforce careful checking of answers.

CONCLUSION

Based on the study of junior high school students regarding the Pythagorean Theorem, students' mathematical reasoning abilities can be classified into three categories: high, medium, and low. The results indicate that the majority of students fall into the low category, experiencing significant difficulties in proposing conjectures, performing mathematical transformations, providing logical justifications, and drawing conclusions.

Students in the medium category were able to reason and solve problems, but their approach lacked consistency. Errors often occurred when they attempted to transform the problem into an accurate mathematical model or provide valid justification for their solutions. On the other hand, students in the high category demonstrated strong reasoning skills, correctly identifying conjectures and applying the Pythagorean Theorem. However, minor mistakes were still observed, mainly due to errors in visualization or insufficient checking of calculations.

Overall, these findings suggest that students' mathematical reasoning skills in the Pythagorean Theorem still need improvement, particularly in solving non-routine problems that require structured and strategic thinking. This limitation may be influenced by learning habits that emphasize memorization and imitation rather than active problem-solving and independent reasoning. Therefore, instructional approaches that focus on conceptual understanding, systematic reasoning, and reflective problem-solving are essential to strengthen students' reasoning abilities. Future research could explore effective interventions to help students systematically solve complex mathematical problems.

ACKNOWLEDGMENTS

The researcher would like to express sincere gratitude to Allah SWT for His grace and guidance, which enabled the completion of this study. This research was supported by Universitas Negeri Surabaya, particularly the Doctoral Program in Mathematics Education, and the researcher is deeply grateful for the invaluable contributions of the faculty and course instructors. Special thanks are extended to SMP Negeri 1 Raha, especially the principal, teachers, and grade VIII students who participated in this study. Their cooperation and willingness to engage in the research greatly contributed to the findings presented in this paper.

The researcher also sincerely thanks family members parents and siblings for their continuous encouragement and motivation throughout this journey. Appreciation is also extended to colleagues and friends who provided support, discussions, and valuable insights that enriched the research process. It is hoped that this study will be beneficial for future research and contribute to the development of mathematical problem-solving education.

REFERENCES

- Absorin, A., & Sugiman, S. (2018). Eksplorasi kemampuan penalaran dan representasi matematis siswa sekolah menengah pertama. *PYTHAGORAS: Jurnal Matematika dan Pendidikan Matematika*, 13(2), 189–202. <https://doi.org/10.21831/pg.v13i2.21249>
- Ariati, C., & Juandi, D. (2022). Kemampuan penalaran matematis: Systematic literature review. *LEMMA: Letters of Mathematics Education*, 8(2), 61–75.

- Astriani, N., & Al Dhana, M. B. (2024). Kemampuan penalaran matematis siswa melalui pendekatan Contextual Teaching and Learning. *Jurnal THEOREMS*, 8(2), 7404. <https://doi.org/10.31949/th.v8i2.7404>
- Khainingsih, F. G., Setiawan, M., & Rahayu, A. (2025). Analisis kemampuan penalaran matematis siswa dalam menyelesaikan soal open-ended pada materi Teorema Pythagoras. *Jurnal Kependidikan*, 6(2), 266–274.
- Kurniawati, I., Hadi, S., & Retnawati, H. (2022). Reflective thinking and its relationship with students' mathematical reasoning ability. *International Journal of Instruction*, 15(2), 89–104.
- Millah, S., & Hidayah, I. (2024). Students' mathematical reasoning ability in solving geometry problems based on cognitive levels. *Journal of Mathematics Education Research*, 8(1), 45–56.
- Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2020). TIMSS 2019 international results in mathematics and science. *International Association for the Evaluation of Educational Achievement*.
- Mulyani, M., Nurjaman, A., & Hendriana, H. (2022). Analisis kemampuan penalaran matematis siswa kelas VIII ditinjau dari motivasi belajar pada materi Teorema Pythagoras. *JPMI: Jurnal Pembelajaran Matematika Inovatif*, 8(2), 24704.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. NCTM.
- Nuraini, N., Subanji, & Susanto, H. (2021). Students' difficulties in conjecturing and reasoning in geometry learning. *Journal of Research in Mathematics Education*, 10(3), 275–289.
- Nurlinda, E., Azis, Z., & Nasution, M. D. (2024). *Students' mathematical reasoning ability and self-efficacy viewed from the application of problem based learning and contextual teaching and learning models assisted*. *Journal of Mathematics Education and Application*, 3(2). <https://doi.org/10.30596/jmea.v3i2.20329>
- Nurwita, F., Kusumah, Y. S., & Priatna, N. (2022). Exploring students' mathematical computational thinking ability in solving Pythagorean Theorem problems. *Al-Jabar: Jurnal Pendidikan Matematika*, 13(2), 273–287. <https://doi.org/10.24042/ajpm.v13i2.12496>
- OECD. (2019). *PISA 2018 results (Volume I): What students know and can do*. OECD Publishing.
- OECD. (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*. OECD Publishing.
- Polya, G. (1945). *How to Solve It: A New Aspect of Mathematical Method*. Princeton University Press.
- Ramdan, G. A., & Roesdiana, L. (2023). Analisis kemampuan penalaran matematis siswa pada materi Teorema Pythagoras. *Educatio FKIP UNMA*, 8(1), 1996. <https://doi.org/10.31949/educatio.v8i1.1996>
- Ramdan, Z. M., et al. (2023). Analysis of students' mathematical reasoning ability in solving geometry problems. *Journal of Mathematical Learning and Instruction*, 6(2), 89–98.
- Rosa, K. W., Nurhanurawati, N., & Suryadinata, N. (2025). Students' mathematical reflective thinking ability with guided discovery learning on Pythagorean Theorem material. *Jurnal Pendidikan Matematika dan Sains*, 13(1), 132–139. <https://doi.org/10.21831/jpms.v13i1.84650>

- Santoso, E., et al. (2023). Mathematical reasoning skills of junior high school students in geometry problem solving. *Journal of Innovative Mathematics Learning*, 6(3), 233–242.
- Sari, M., Nafisah, Z., & Sihotang, R. D. (2024). Analysis of students' ability to solve mathematical problems in solving Pythagorean Theorem. *Riemann: Research of Mathematics and Mathematics Education*, 6(3), 44. <https://doi.org/10.38114/reimann.v6i3.44>
- Sari, R. N. (2020). Peningkatan kemampuan penalaran matematis siswa SMP melalui pembelajaran penemuan terbimbing. *PYTHAGORAS: Jurnal Program Studi Pendidikan Matematika*, 4(2), 189.
- Shafira, Z. N., Adetia, E., Julaeha, N. S., Pertiwi, Y. A., & Adz Zikri, M. F. (2025). Tren penelitian kemampuan penalaran matematis di Indonesia. *Intellectual Mathematics Education*, 1(1).
- Stylianides, A. J., & Stylianides, G. J. (2018). Classroom practices with mathematical argumentation: A review of research. *Educational Studies in Mathematics*, 98(1), 1–30.
- Sudijono, A. (2011). *Pengantar statistik pendidikan*. Jakarta: RajaGrafindo Persada.
- Wulandari, S., Syahbana, A., Tanzimah, T., Shang, Y., & Weinhandl, R. (2024). Analysis of students' thinking level in solving Pythagoras' theorem problems based on Van Hiele's theory. *Malikussaleh Journal of Mathematics Learning*, 4(2), 3905. <https://doi.org/10.29103/mjml.v4i2.3905>