
STUDENTS' METACOGNITIVE ERRORS BASED ON NEWMAN'S ERROR TYPES WITHIN DEEP LEARNING APPROACH

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ARTICLE INFO

Article history:

Received Jan 27, 2026

Revised Feb 06, 2026

Accepted Feb 23, 2026

Keywords:

Deep Learning

Metacognitive Ability

Newman's Error Analysis

ABSTRACT

Students' difficulties in solving mathematical problems are often closely related to weaknesses in metacognitive regulation, particularly in planning, monitoring, and evaluating problem-solving processes. One systematic framework to identify these difficulties is Newman's Error Analysis, which classifies students' errors into sequential stages of problem solving. This study aims to describe students' metacognitive errors based on Newman's error types in mathematics learning using a Deep Learning approach. This research employed a mixed methods approach with a sequential explanatory design, focusing on qualitative descriptive analysis. The participants consisted of three ninth-grade students from SMP Islamiyah Ciawi in the 2025/2026 academic year, selected purposively to represent high, moderate, and low levels of metacognitive ability. Data were collected through open-ended problem-solving tests on solid figures, a metacognitive questionnaire using a Likert scale, and semi-structured interviews. Data analysis was conducted by identifying students' errors at each stage of Newman's procedure—reading, comprehension, transformation, process skills, and encoding—and relating them to metacognitive indicators. Methodological triangulation was applied to enhance the credibility of the findings. The results indicate that students with high metacognitive ability tend to exhibit minimal errors, mainly at the encoding stage. In contrast, students with moderate and low metacognitive abilities demonstrate dominant errors at the transformation and process skills stages, with low-metacognitive students also experiencing reading and comprehension errors. These findings suggest that metacognitive regulation significantly influences the type and stage of students' errors. In conclusion, integrating explicit metacognitive scaffolding within Deep Learning practices is essential to reduce students' errors and enhance mathematical problem-solving performance.

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How to Cite:

Astuti, W., Lestari, P., & Prabawati, M. N. (2026). Students' Metacognitive Errors based on Newman's Error Types within Deep Learning Approach. *JIML*, 9(1), 191-203.

INTRODUCTION

Mathematics learning is not solely concerned with the acquisition of conceptual knowledge and procedural fluency; rather, it also requires the development of higher-order thinking skills that involve awareness, regulation, and control of one's own cognitive processes. These abilities are commonly referred to as metacognition, which encompasses planning, monitoring, evaluation, and reflection during problem-solving activities (Flavell, 1979; Schraw & Dennison, 1994). Students with well-developed metacognitive skills tend to demonstrate greater flexibility in selecting strategies, higher accuracy in executing procedures, and stronger abilities to detect and correct errors during mathematical problem solving (Zimmerman, 2002; Desoete, 2007; Stillman, 2011).

Extensive research has demonstrated that metacognition plays a critical role in students' success in mathematics, particularly in non-routine problem-solving contexts that require reasoning, representation, and decision making (Schoenfeld, 1992; Garofalo & Lester, 1985; Laurens et al., 2018). Students who actively engage in metacognitive processes are more capable of understanding problem situations, planning appropriate solution strategies, monitoring their progress, and evaluating the correctness of their solutions (Hacker, Dunlosky, & Graesser, 2009; Özsoy & Ataman, 2017). Conversely, limited metacognitive awareness often leads students to rely on trial-and-error procedures, apply formulas mechanically, and overlook inconsistencies in their reasoning.

Despite its importance, numerous empirical studies report that students continue to experience substantial difficulties in solving mathematical problems, particularly in geometry-related topics such as solid figures. Geometry requires not only conceptual understanding but also spatial reasoning, representation skills, and the ability to translate verbal information into mathematical models (Battista, 2007; Clements & Sarama, 2011). In the context of solid figures, students frequently struggle to interpret problem statements, identify relevant information, construct appropriate models, and perform accurate calculations (Jones, 2002; Gutiérrez, 2013). These difficulties often result in persistent errors that reflect deeper issues related to students' metacognitive regulation rather than merely conceptual gaps.

Errors in mathematical problem solving are not inherently negative; instead, they can provide valuable insights into students' thinking processes when analyzed systematically (Radatz, 1979; Borasi, 1994). Error analysis allows researchers and educators to identify where and why students fail during problem solving, thereby offering a foundation for targeted instructional interventions. One well-established framework for analyzing students' errors is Newman's Error Analysis (NEA), which categorizes errors into five sequential stages: reading, comprehension, transformation, process skills, and encoding (Newman, 1977; White, 2010).

Newman's procedure has been widely applied in mathematics education research to diagnose students' problem-solving difficulties across different mathematical domains and educational levels (Singh, Rahman, & Hoon, 2010; Clement, 1982; Praktipong & Nakamura, 2006). The framework is particularly useful because it aligns closely with the cognitive and metacognitive stages involved in problem solving. For instance, errors at the reading and comprehension stages indicate difficulties in understanding the problem context, whereas errors at the transformation and process skills stages reflect challenges in planning and monitoring solution strategies. Encoding errors, on the other hand, are often associated with weaknesses in evaluation and reflection processes (Polya, 1973; Schoenfeld, 1985).

Several studies have highlighted strong relationships between Newman's error types and students' metacognitive abilities. Students with low metacognitive awareness tend to commit multiple errors across several Newman stages, particularly at the transformation and process skills levels, due to ineffective planning and monitoring (Amir et al., 2019; Zakaria & Maat,

2010). In contrast, students with higher metacognitive skills typically demonstrate fewer errors and are more capable of self-correcting mistakes, especially during the later stages of problem solving (Kramarski & Mevarech, 2003; Özcan & Eren Gümüş, 2019).

In recent years, the Deep Learning approach in education has gained increasing attention as a pedagogical paradigm that emphasizes meaningful learning, conceptual understanding, critical thinking, and reflective practice (Fullan, Quinn, & McEachen, 2018; Trilling & Fadel, 2009). In mathematics education, Deep Learning encourages students to actively construct knowledge, connect concepts across contexts, and engage in reflective thinking rather than memorizing procedures (Hattie & Donoghue, 2016; Darling-Hammond et al., 2020). These characteristics make Deep Learning highly compatible with the development of metacognitive skills.

Deep Learning environments typically promote inquiry-based learning, problem-based learning, and reflective dialogue, all of which support students' metacognitive engagement (Biggs & Tang, 2011; Prince & Felder, 2006). Through meaningful tasks and reflective questioning, students are encouraged to articulate their thinking, monitor their understanding, and evaluate their solutions. However, empirical evidence suggests that the implementation of Deep Learning alone does not automatically eliminate students' errors; without explicit metacognitive scaffolding, students may still struggle at critical stages of problem solving, particularly during transformation and procedural execution (Kirschner, Sweller, & Clark, 2006; Chen & Yang, 2023).

Given these considerations, analyzing students' metacognitive errors using Newman's framework within a Deep Learning context becomes highly relevant. Such analysis provides a nuanced understanding of how students with different levels of metacognitive ability navigate problem-solving processes and where breakdowns occur. Moreover, focusing on a small number of purposively selected students allows for an in-depth exploration of individual thinking patterns, which is essential for qualitative and mixed-methods research in mathematics education (Creswell & Plano Clark, 2018; Miles, Huberman, & Saldaña, 2014).

Therefore, this study aims to describe students' metacognitive errors based on Newman's error types in mathematics learning using a Deep Learning approach. By examining students with high, moderate, and low metacognitive abilities, this study seeks to contribute to a deeper understanding of the relationship between metacognition and error patterns in mathematical problem solving. The findings are expected to provide theoretical and practical implications for designing instructional strategies that explicitly foster metacognitive regulation and reduce persistent errors in mathematics learning.

METHOD

This study employed a mixed methods approach with a sequential explanatory design, in which quantitative data were collected and analyzed in the initial phase, followed by qualitative data to explain and elaborate the quantitative findings (Creswell & Plano Clark, 2018). This design was selected to provide a comprehensive understanding of students' metacognitive errors by integrating numerical measurement of metacognitive ability with in-depth qualitative analysis of students' problem-solving processes. However, this article specifically focuses on the qualitative descriptive phase of the study, which aims to explore students' metacognitive errors based on Newman's error framework.

The research participants consisted of three ninth-grade students from SMP Islamiyah Ciawi in the 2025/2026 academic year. Participants were selected using purposive sampling, a technique commonly employed in qualitative research to obtain information-rich cases that represent specific characteristics relevant to the research objectives (Patton, 2015). The three students were chosen to represent high, moderate, and low levels of metacognitive ability, as determined

by the results of a metacognitive questionnaire administered prior to the qualitative analysis. This sampling strategy allowed for an in-depth comparison of error patterns across different metacognitive profiles (Miles, Huberman, & Saldaña, 2014).

Data were collected using three primary research instruments. First, an open-ended mathematical problem-solving test was administered on the topic of solid figures. Open-ended items are effective for revealing students' reasoning processes and metacognitive regulation during problem solving (Schoenfeld, 1985; Polya, 1973). Second, a metacognitive questionnaire using a four-point Likert scale was employed to assess students' metacognitive abilities across several indicators, including planning, understanding, monitoring, evaluation, and reflection (Schraw & Dennison, 1994; Desoete, 2007). Third, semi-structured interviews were conducted to explore students' thinking processes, strategy selection, and self-regulation in greater depth, as this interview format allows flexibility while maintaining alignment with the research focus (Cohen, Manion, & Morrison, 2018).

Data analysis was conducted by systematically examining students' written responses to identify errors at each stage of Newman's Error Analysis, which classifies problem-solving errors into five sequential stages: reading, comprehension, transformation, process skills, and encoding (Newman, 1977; White, 2010). This framework has been widely used to diagnose students' difficulties in mathematical problem solving and to reveal underlying cognitive and metacognitive weaknesses (Prakitipong & Nakamura, 2006; Singh, Rahman, & Hoon, 2010). Each identified error was coded according to the corresponding Newman stage and then analyzed in relation to students' metacognitive indicators.

To enhance the credibility and trustworthiness of the findings, the results of the written test analysis were triangulated with data from the metacognitive questionnaire and interview transcripts. Methodological triangulation is a well-established strategy in qualitative and mixed methods research to improve validity by cross-verifying findings across multiple data sources (Denzin, 1978; Creswell, 2014). Through this triangulation process, a more comprehensive and accurate understanding of students' metacognitive errors in mathematics learning was achieved.

RESULTS AND DISCUSSION

Results

This section presents the results of the analysis of students' metacognitive errors based on Newman's Error Analysis framework. The findings are organized according to students' metacognitive ability levels high, moderate, and low and are supported by tabular data derived from written test analysis and interview confirmation.

1. Profile of Research Participants

Based on the metacognitive questionnaire results, three students were purposively selected to represent different levels of metacognitive ability. Table 1 summarizes the profile of the research participants.

Table 1. Profile of Research Participants Based on Metacognitive Ability

Student Code	Metacognitive Level	General Characteristics
SMT	High	Able to plan strategies, monitor procedures, and evaluate results
SMS	Moderate	Understands problems but lacks consistent monitoring and evaluation
SMR	Low	Does not plan systematically and rarely reflects on solutions

The classification indicates clear differences in students' awareness and regulation of their cognitive processes during mathematical problem solving.

2. Types of Errors Based on Newman's Error Analysis

Students' written responses were analyzed using Newman's five-stage error framework. Each stage was examined to identify whether a specific type of error occurred. The results of this analysis are presented in Table 2.

Table 2. Metacognitive Errors Based on Newman's Error Types for Each Student

Student	Reading	Comprehension	Transformation	Process Skills	Encoding
SMT	No	No	No	No	Yes
SMS	No	Yes	Yes	Yes	Yes
SMR	Yes	Yes	Yes	Yes	No

Table 2 shows that the pattern of errors varies according to students' metacognitive ability levels. Students with higher metacognitive ability tend to exhibit fewer errors, while those with moderate and low metacognitive abilities demonstrate multiple errors across several Newman stages.

3. Analysis of Errors by Newman Stage

a. Reading Errors

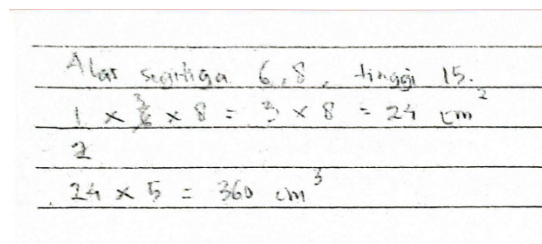


Figure 1. Reading Errors SMR

Reading errors occurred only in the low-metacognitive student (SMR) based on figure 1. This student misread numerical information and mathematical symbols, leading to incorrect interpretation of the problem statement. In contrast, students with high and moderate metacognitive abilities were able to read the problem accurately without difficulty. This finding suggests that weak metacognitive planning and attention control may contribute to early-stage failures in problem solving.

b. Comprehension Errors



Figure 2. Comprehension Errors SMR

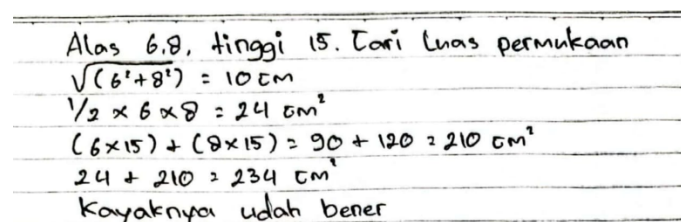


Figure 3. Comprehension Errors SMS

Comprehension errors were identified in students with moderate and low metacognitive abilities (SMS and SMR) based on Figure 2 and Figure 3. These students were unable to clearly identify what was known and what was asked in the problem. Interview data confirmed that both students tended to proceed directly to calculations without fully understanding the problem context. The high-metacognitive student (SMT) demonstrated the ability to restate the problem in their own words, indicating strong understanding and awareness at this stage.

c. Transformation Errors

Transformation errors were found in both moderate- and low-metacognitive students based on Figure 4 and Figure 5 below. These errors involved difficulties in converting verbal problem statements into appropriate mathematical models or selecting relevant formulas.

$$6 \times 8 \times 15 = 720 \text{ cm}^3$$

Figure 4. Transformation Errors SMR

$$\begin{aligned} &\text{Alas } 6,8, \text{ tinggi } 15. \text{ Cari luas permukaan} \\ &\sqrt{(6^2 + 8^2)} = 10 \text{ cm} \\ &\frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2 \\ &(6 \times 15) + (8 \times 15) = 90 + 120 = 210 \text{ cm}^2 \\ &24 + 210 = 234 \text{ cm}^2 \\ &\text{Kayaknya udah benar} \end{aligned}$$

Figure 5. Transformation Errors SMS

Table 3. Transformation Errors and Metacognitive Indicators

Student	Observed Error	Related Metacognitive Weakness
SMS	Incorrect formula selection	Inadequate planning
SMR	No mathematical model constructed	Lack of planning and understanding

d. Process Skills Errors

Process skills errors were dominant among students with moderate and low metacognitive abilities. These errors included incorrect calculations, incomplete procedures, and failure to monitor intermediate results.

$$\begin{aligned} &\text{Sisi alas } 60 \text{ cm, tinggi limas } 40 \text{ cm} \\ &\text{Cari volume} \\ &\text{Bersamaan limas di dalam prisma persegi yang alasnya sama,} \\ &\text{tingginya sama.} \\ &V = s \times s \times t \\ &= 60 \times 60 \times 40 = 144.000 \text{ cm}^3 \\ &V \text{ limas} = \frac{1}{3} \times V \text{ prisma} = 144.000 : 3 = 48.000 \text{ cm}^3 \\ &\text{Rumus Limas} = \frac{1}{3} \times \text{La} \times t \\ &= \frac{1}{3} \times (60 \times 60) \times 40 \\ &= \frac{1}{3} \times 144.000 = 48.000 \text{ sesuai} \\ &\text{Konsep prisma membantu, ingat perubahan posisi} \end{aligned}$$

Figure 6. Process Skills Errors SMT

alas 60cm, tinggi 40cm
 $60 \times 60 \times 40 = 144.000 \text{ cm}^2$
 $\frac{144.000}{2} = 72.000 \text{ cm}^2$
 Sepertinya benar
 Lupa bukannya berapa

Figure 7. Process Skills Errors SMS

alas 60, tinggi 40
 $60 \times 60 = 3600 \text{ cm}^2$
 $3600 \times 40 = 144.000 \text{ cm}^3$
 tidak paham maksudnya.

Figure 8. Process Skills Errors SMR

Table 4. Process Skills Errors Across Metacognitive Levels

Student	Type of Error	Description
SMT (Figure 6)	None	Procedures executed accurately
SMS (Figure 7)	Calculation error	Failed to recheck arithmetic operations
SMR (Figure 8)	Procedural error	Steps performed randomly without sequence

The absence of process skills errors in the high-metacognitive student highlights the importance of monitoring and self-regulation during mathematical procedures.

e. Encoding Errors

Encoding errors were identified in students with high and moderate metacognitive abilities. These errors involved incorrect final answers or incomplete conclusions, despite relatively correct procedures. Interestingly, the low-metacognitive student did not exhibit encoding errors, as incorrect solutions had already occurred at earlier stages. This suggests that encoding errors may emerge only when earlier problem-solving stages are partially successful.

4. Relationship Between Metacognitive Ability and Error Patterns

To further clarify the relationship between metacognitive ability and error occurrence, the distribution of errors across Newman stages is summarized in Table 5.

Table 5. Distribution of Newman Error Types by Metacognitive Level

Metacognitive Level	Dominant Error Types
High	Encoding (minor)
Moderate	Transformation, Process Skills
Low	Reading, Comprehension, Transformation, Process Skills

The table indicates a clear progression: as metacognitive ability decreases, the number and complexity of errors increase and appear earlier in the problem-solving process.

5. Summary of Results

Overall, the results demonstrate that students' metacognitive ability strongly influences both the type and stage of errors encountered during mathematical problem solving. High-metacognitive students exhibit strong control over early and middle problem-solving stages,

while moderate- and low-metacognitive students struggle particularly with transformation and process skills stages. These findings confirm that metacognitive regulation plays a critical role in minimizing errors and supporting successful problem solving in mathematics learning.

Discussions

The findings of this study reveal a clear relationship between students' metacognitive ability levels and the types of errors they commit during mathematical problem solving, as classified by Newman's Error Analysis. The results indicate that students with high metacognitive ability tend to exhibit minimal errors, primarily at the encoding stage, whereas students with moderate and low metacognitive abilities demonstrate dominant errors at the transformation and process skills stages. These patterns provide important insights into how metacognitive regulation influences students' problem-solving performance in mathematics.

The absence of reading and comprehension errors among students with high metacognitive ability suggests that these students possess strong metacognitive planning and awareness at the initial stages of problem solving. This finding aligns with Flavell's (1979) theory of metacognition, which emphasizes that metacognitively skilled learners are more capable of activating prior knowledge, focusing attention, and understanding task demands. Similarly, Schraw and Dennison (1994) argue that students with high metacognitive awareness are more effective in interpreting problem statements and identifying relevant information, thereby reducing the likelihood of early-stage errors.

In contrast, the presence of reading and comprehension errors among students with low metacognitive ability indicates a lack of cognitive monitoring and task awareness. These students tended to misinterpret symbols, overlook key information, and fail to identify what was being asked in the problem. Such findings are consistent with previous studies reporting that students with limited metacognitive skills often rush into calculations without fully understanding the problem context (Schoenfeld, 1985; Garofalo & Lester, 1985). This behavior reflects weak planning processes and insufficient self-questioning, which are essential components of effective metacognitive regulation.

One of the most prominent findings of this study is the dominance of transformation errors among students with moderate and low metacognitive abilities. Transformation errors occurred when students failed to translate verbal problem statements into appropriate mathematical models or selected incorrect formulas. This result supports earlier research indicating that transformation is a critical and cognitively demanding stage in mathematical problem solving, as it requires both conceptual understanding and strategic planning (Polya, 1973; Newman, 1977; Clement, 1982). Students who lack metacognitive planning skills often struggle to determine suitable solution strategies, leading to incorrect or incomplete mathematical representations.

The findings further corroborate the work of Prakitipong and Nakamura (2006) and Singh et al. (2010), who reported that transformation errors are among the most frequent errors in mathematics problem solving, particularly among students with lower levels of self-regulation. From a metacognitive perspective, these errors indicate a failure to plan effectively, as students are unable to anticipate the steps required to solve the problem or to evaluate whether their chosen strategy is appropriate (Zimmerman, 2002; Desoete, 2007).

Process skills errors were also found to be dominant among students with moderate and low metacognitive abilities. These errors involved incorrect calculations, disorganized procedures, and a lack of consistency in following solution steps. Such findings highlight the importance of metacognitive monitoring during mathematical procedures. According to Schoenfeld (1992), effective problem solvers continuously monitor their progress, check intermediate results, and

adjust strategies when errors are detected. Students who lack these monitoring skills are more likely to persist with incorrect procedures and fail to recognize computational mistakes.

The results of this study are consistent with Kramarski and Mevarech (2003), who found that students with weak metacognitive monitoring skills tend to commit repeated procedural errors, even when they possess adequate conceptual knowledge. Similarly, Özsoy and Ataman (2017) emphasize that monitoring and self-regulation play a crucial role in ensuring procedural accuracy, particularly in multi-step mathematical problems such as those involving solid figures.

Interestingly, encoding errors were observed among students with high and moderate metacognitive abilities, while the low-metacognitive student did not exhibit encoding errors. This finding suggests that encoding errors tend to occur only when students successfully progress through earlier stages of problem solving. In other words, encoding errors may reflect incomplete evaluation and reflection rather than a total breakdown of problem-solving processes. This interpretation aligns with the work of White (2010), who argues that encoding errors often occur when students fail to verify whether their final answers are reasonable or aligned with the problem context.

From a metacognitive standpoint, encoding errors indicate weaknesses in the evaluation and reflection components of metacognition (Schraw & Moshman, 1995; Hacker et al., 2009). Even students with relatively strong planning and monitoring skills may neglect to re-express answers clearly or to check the validity of their conclusions. This finding underscores the need for instructional practices that explicitly encourage students to reflect on their solutions and articulate final answers accurately.

The implementation of the Deep Learning approach in this study provides an important contextual lens for interpreting the results. Deep Learning emphasizes meaningful learning, conceptual understanding, and reflective thinking, all of which are closely aligned with metacognitive development (Biggs & Tang, 2011; Fullan, Quinn, & McEachen, 2018). The relatively strong performance of the high-metacognitive student suggests that Deep Learning environments may support students who already possess metacognitive awareness by providing opportunities for reflection and conceptual integration.

However, the persistence of transformation and process skills errors among students with moderate and low metacognitive abilities indicates that Deep Learning alone may not be sufficient to reduce metacognitive errors without explicit scaffolding. This finding supports the argument of Kirschner, Sweller, and Clark (2006), who caution that minimally guided instructional approaches may be less effective for learners with limited self-regulation skills. Without structured guidance, these students may struggle to manage cognitive load and regulate their problem-solving processes effectively.

Several studies have emphasized the importance of integrating explicit metacognitive instruction within student-centered learning approaches (Hattie & Donoghue, 2016; Darling-Hammond et al., 2020). Strategies such as guided questioning, self-explanation prompts, and reflective journals can help students develop planning, monitoring, and evaluation skills. In the context of Newman's Error Analysis, such strategies may be particularly effective in reducing transformation and process skills errors by encouraging students to think critically about their chosen strategies and procedures.

The qualitative focus on three purposively selected students in this study allows for a deeper understanding of individual error patterns and metacognitive processes. While the small sample size limits generalizability, it provides rich insights into how students with different metacognitive profiles experience and regulate mathematical problem solving. This approach

is consistent with qualitative research principles that prioritize depth of understanding over breadth (Miles, Huberman, & Saldaña, 2014; Creswell & Plano Clark, 2018).

Overall, the findings of this study reinforce the theoretical and empirical link between metacognition and mathematical problem solving. Students' errors are not merely procedural or conceptual in nature but are deeply connected to their ability to regulate cognitive processes throughout the problem-solving cycle. By combining Newman's Error Analysis with a Deep Learning framework, this study contributes to a more nuanced understanding of how instructional approaches and metacognitive abilities interact to influence students' mathematical performance.

CONCLUSION

This study investigated students' metacognitive errors in mathematics problem solving based on Newman's Error Analysis within a Deep Learning approach. By focusing on three ninth-grade students representing high, moderate, and low levels of metacognitive ability, the study provided an in-depth qualitative description of how metacognitive regulation influences error patterns at different stages of problem solving.

The findings indicate that students with high metacognitive ability experience minimal errors, primarily at the encoding stage, reflecting minor weaknesses in evaluation and reflection rather than fundamental misunderstandings. In contrast, students with moderate and low metacognitive abilities demonstrate dominant errors at the transformation and process skills stages, as well as earlier-stage errors in reading and comprehension for low-metacognitive students. These patterns suggest that inadequate planning and monitoring are key contributors to persistent errors in mathematical problem solving.

Overall, the results confirm that students' errors are not merely procedural or conceptual but are closely linked to their metacognitive regulation throughout the problem-solving process. While the Deep Learning approach supports meaningful and reflective learning, its effectiveness depends on the extent to which metacognitive skills are explicitly developed and scaffolded. Without deliberate support, students with limited metacognitive awareness may continue to experience difficulties, particularly in transforming problems into mathematical models and monitoring procedural accuracy.

In conclusion, this study highlights the importance of integrating explicit metacognitive instruction with Deep Learning practices in mathematics education. By addressing students' metacognitive weaknesses at specific Newman stages, educators can design more targeted instructional strategies to reduce errors and enhance students' problem-solving performance. The findings contribute to a deeper understanding of the interplay between metacognition, error patterns, and instructional approaches, offering valuable insights for both theory and practice in mathematics education.

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