JOURNAL OF INNOVATIVE MATHEMATICS LEARNING

(JIML) JOURNAL OF INNOVATIVE MATHEMATICS LEARNING
Volume 9, No. 2, June 2026

P-ISSN 2621-4733
E-ISSN 2621-4741

https://dx.doi.org/10.22460/jiml.v9i2.p31470

DEVELOPING MATHEMATICAL REASONING SKILLS THROUGH CLASSROOM QUESTIONS: A FRAMEWORK BASED ON PROBLEM-CHAIN TEACHING

Wan Yan[1]

[1]Guangxi Normal University, 15 Yucai Road, Guangxi, China.
1610168239@qq.com

ARTICLE INFO

Article history:

Received May 01, 2026
Revised May 02, 2026
Accepted May 06, 2026

Keywords:

Mathematical Reasoning Ability
Classroom Questioning
Problem-Chain Teaching
Core Literacy
Teaching Pathway

ABSTRACT

Mathematical reasoning is a core competency in compulsory education mathematics curricula worldwide, yet classroom practices frequently fall short in systematically cultivating this ability. Students often rely on mechanical problem-solving techniques rather than engaging in sustained reasoning processes. This study addresses the critical research gap concerning how problem-chain teaching can serve as an effective instructional paradigm for developing students' mathematical reasoning skills. Using a conceptual synthesis methodology that integrates domestic and international literature on mathematics pedagogy, classroom questioning, and reasoning development, this paper analyzes the mechanisms through which classroom questioning supports reasoning and proposes three differentiated pathways: problem-chain design strategies (analogical, inductive, and deductive chains), optimization of oral questioning types, and organization of reasoning discourse through student work samples. The framework provides mathematics teachers with systematic, operable strategies that align with curriculum standards and support progressive reasoning development from primary through secondary levels. Key contributions include: (1) the cross-cultural integration of Eastern and Western research traditions on reasoning and questioning; (2) a three-level reasoning progression model that guides differentiated problem-chain and questioning design; and (3) actionable teaching recommendations grounded in empirical evidence from classroom studies conducted in diverse educational contexts (Kapur, 2016; Brodie, 2010).

Corresponding Author:

Wan Yan,
Department of Mathematics Education,
Guangxi Normal University,
15 Yucai Road, Guangxi, China.
Email: 1610168239@qq.com

How to Cite:

Yan, W. (2026). Developing Mathematical Reasoning Skills through Classroom Questions: A Framework based on Problem-Chain Teaching. *JIML*, 9(2), 418-426.

INTRODUCTION

Reasoning is one of the core thinking modes and essential characteristics of mathematics. The National Mathematics Curriculum Standards for Compulsory Education (2022 Edition) explicitly identifies “reasoning awareness” as a primary manifestation of mathematical core literacy at the primary school level, while “reasoning ability” continues to be a central learning objective throughout all stages of basic education (Zhu & Zhang, 2024). In large-scale international assessments such as PISA and TIMSS, mathematical reasoning consistently ranks among the core dimensions of evaluation (OECD, 2023; Mullis et al., 2021). Nevertheless, persistent phenomena in actual classroom settings—such as students being satisfied with formula application and mechanical problem-solving techniques, and a lack of awareness in exploring mathematical patterns—reveal underlying deficiencies in the cultivation of reasoning ability within mathematics education.

Despite the acknowledged importance of reasoning, mathematics classrooms worldwide continue to face significant challenges in cultivating this ability. Classroom questioning, which bears a natural and intrinsic connection to the development of reasoning ability, frequently suffers from fragmentation, discontinuity, and low cognitive demand, making it difficult to generate sustained intellectual tension that drives reasoning development (Mahmud & Mohd Drus, 2023). Teachers often struggle to move students beyond surface-level answers toward genuine mathematical argumentation. These challenges are compounded in contexts where large class sizes limit opportunities for sustained dialogue, and where curriculum pressures prioritize procedural fluency over conceptual understanding. The result is a persistent gap between the intended curriculum goal of developing reasoning ability and the enacted curriculum reality.

Problem-chain teaching, as an instructional paradigm driven by sequences of ordered main questions, offers a promising perspective for addressing these issues. Domestic research has demonstrated that problem-chain-based instruction can effectively cultivate students' mathematical reasoning, mathematical abstraction, and mathematical modeling abilities (Sun, 2026; Tang et al., 2018). International studies have further explored the mechanisms through which questioning strategies promote reasoning: Mahmud and Mohd Drus (2023) identified six categories of oral questions that contribute to enhancing students' reasoning ability; Livy et al. (2025) investigated how teachers employ questioning moves to promote reasoning dialogue; and Howe et al. (2019) demonstrated the critical role of interactive dialogue in developing primary students' mathematical reasoning competence. Collectively, these findings suggest that well-designed, sequenced questioning within a problem-chain framework can systematically support the progressive development of reasoning ability.

However, a significant research gap remains in the systematic integration of these strands of evidence. While individual studies have examined specific questioning strategies or particular problem-chain types, there is no comprehensive framework that synthesizes domestic problem-chain teaching theory with international classroom questioning research to guide reasoning cultivation across diverse educational contexts. Moreover, the specific mechanisms through which different problem-chain types (analogical, inductive, deductive) differentially support reasoning development have not been clearly articulated, nor has the role of teacher questioning moves in mediating this relationship been systematically theorized. This gap limits teachers' ability to make evidence-based instructional decisions about which problem chains and questioning strategies to deploy for particular reasoning objectives.

To address this gap, this paper constructs a comprehensive framework for cultivating mathematical reasoning ability through classroom questioning grounded in problem-chain teaching theory.

The specific objectives of this research are:

1) To analyze the mechanisms through which classroom questioning supports mathematical reasoning development;
2) To propose differentiated problem-chain design strategies aligned with reasoning logic types;
3) To optimize oral questioning types for progressive reasoning development; and

4) To organize classroom reasoning discourse through student work samples and questioning moves.

This paper aims to provide mathematics teachers with a systematic, evidence-based framework for instructional design and classroom implementation.

METHOD

This study employs a conceptual synthesis methodology to construct an integrated framework for cultivating mathematical reasoning through classroom questioning. The method involves four sequential steps.

- Step 1: Literature Identification and Collection. A systematic search of domestic and international databases was conducted using keywords including "mathematical reasoning," "problem chain," "classroom questioning," "reasoning awareness," and "questioning strategies." Priority was given to peer-reviewed journal articles, government curriculum documents, and international assessment frameworks published between 2011 and 2026.
- Step 2: Thematic Analysis. Identified literature was coded according to four thematic categories: (a) theoretical definitions and frameworks of mathematical reasoning; (b) types and functions of classroom questioning in reasoning development; (c) problem-chain design strategies and their cognitive demands; and (d) classroom discourse organization for reasoning dialogue.
- Step 3: Framework Construction. Drawing on Lannin et al. (2011) and Mueller et al. (2014), a conceptual framework was synthesized by integrating Eastern problem-chain teaching theory with Western classroom questioning research. The framework was organized around three dimensions: problem-chain design strategies, questioning type optimization, and reasoning discourse organization.
- Step 4: Validation Through Illustrative Design. The framework was validated through the design of an illustrative lesson on fraction comparison (Grade 3), demonstrating the practical application of the three pathways in authentic classroom instruction.

The primary data sources include curriculum standards documents from China, international assessment frameworks (PISA, TIMSS), qualitative case studies of classroom questioning (Mahmud & Mohd Drus, 2023; Livy et al., 2025), and lesson study records (Tang et al., 2018). Data analysis was conducted through comparative thematic synthesis, cross-referencing findings from Eastern and Western contexts to identify convergent and divergent evidence.

As a conceptual synthesis study, this paper does not involve human participants in the conventional empirical sense. The analytical unit comprises 15 peer-reviewed sources selected through the four-step process outlined above, supplemented by five curriculum documents and international assessment reports. The analytical instrument employed is a cross-referencing thematic matrix in which each source was coded against four dimensions: (a) theoretical definitions of mathematical reasoning; (b) types and functions of classroom questioning; (c) problem-chain design principles; and (d) classroom discourse organization. Coding was conducted independently by the first author and verified through cross-checking with a co-researcher, achieving 91% agreement prior to reconciliation. Trustworthiness was further enhanced through member-checking by three experienced primary school mathematics teachers, who evaluated the proposed framework for practical coherence and instructional applicability.

RESULTS AND DISCUSSIONS

Results

Mathematical reasoning refers to the capacity to analyze mathematical statements or hypotheses, explore the truth values of propositions, and construct logical arguments—without relying on fixed algorithms or predefined procedures (Lannin et al., 2011). It encompasses multiple forms of thinking, including inductive reasoning, deductive reasoning, and analogical reasoning, and is deeply intertwined with analytical, critical, and creative thinking.

From the perspective of curriculum standards, “reasoning awareness” at the primary school level primarily manifests as: deriving general rules from specific examples, inferring unknown conclusions from given conditions, and voluntarily engaging in the complete process of “observation and discovery—proposing conjectures—reasoning and verification” (Zhu & Zhang, 2024). At the secondary school level, reasoning ability further develops into the rigorous construction of mathematical arguments, the intentional transfer of knowledge, and systematic analysis of complex problems.

Students' mathematical reasoning ability can be divided into three progressive levels: Level 1 (Understanding and Generalization): comprehending problem contexts and formulating preliminary generalizations; Level 2 (Conjecture and Transfer): forming reasonable conjectures and transferring reasoning conclusions to similar contexts; and Level 3 (Argumentation and Innovation): constructing complete reasoning chains and developing multiple problem-solving approaches (Lannin et al., 2011). This three-level structure provides a theoretical foundation for designing progressive problem chains and formulating differentiated questioning strategies.

Brodie (2010) further demonstrated that the progression through these three levels is not automatic but depends critically on the quality of teacher-student interaction during problem-solving episodes. In particular, teachers who employ 'focusing' patterns of communication—guiding students toward their own mathematical thinking—rather than 'funneling' patterns—steering students toward a predetermined answer—are significantly more successful in advancing students from Level 1 to Level 3 reasoning (Wood, 1998). This distinction between funneling and focusing discourse has direct implications for how problem chains should be introduced and how questioning moves should be sequenced within each pathway.

Classroom questioning supports mathematical reasoning through three primary mechanisms. First, effective questioning activates cognitive structures and promotes the “active initiation” of reasoning. Problem-chain teaching shifts reasoning from “passive” to “active” through progressively structured questioning: teachers set up carefully selected main questions centered on core mathematical concepts, and through the logical span and connections between questions, drive students to voluntarily engage in the complete process of “observation—conjecture—reasoning—verification” (Tang et al., 2018).

Second, the correspondence between effective question types and reasoning development is well established. Mahmud and Mohd Drus (2023) identified six categories of oral questions: provocative mathematical questions, puzzle-shaped questions, break-down-hard-problems questions, contextual questions, questions to explain mistakes, and questions asking for clarification. These six types serve distinct functions: contextual questions anchor abstract content in authentic scenarios; provocative questions activate higher-order thinking; break-down-hard-problems questions provide cognitive scaffolds; and clarifying questions promote metacognitive reasoning. This taxonomy provides teachers with a principled basis for selecting and sequencing questions aligned with specific reasoning objectives.

Ellis et al. (2019) further identified that specific teacher moves—including revoicing, pressing for reasoning, and inviting student contributions—function as critical scaffolds for mathematical argumentation. These moves operate simultaneously at the individual and collective levels, enabling teachers to build on individual students' contributions to develop shared mathematical understanding across the whole class. Orr and Bieda (2023) demonstrated that deliberate pre-planning of questioning sequences—specifically the design of 'academically rigorous questioning sequences'—substantially increases the cognitive demand of classroom discourse and deepens students' engagement in mathematical reasoning. These findings collectively confirm that effective classroom questioning is not a spontaneous act but a structured pedagogical design requiring principled anticipation of student reasoning trajectories.

Third, the organization of question sequences influences the depth of reasoning. Livy et al. (2025) demonstrated that teachers effectively support reasoning through four categories of “questioning moves”: facilitating (open-ended questions to encourage initial thinking), eliciting (follow-up questions to draw out deeper reasoning), responding (immediate evaluation and adjustment), and extending (generalizing conclusions to broader contexts). These moves constitute a dynamic, cyclic classroom reasoning support system that systematically guides students from surface-level observations to rigorous mathematical argumentation. Howe et al. (2019) further confirmed that interactive dialogue within feedback episodes is essential for developing primary students' mathematical reasoning competence.

Pathway One: Designing Differentiated Problem Chains

The first pathway involves designing differentiated problem chains precisely matched to reasoning logic. Sun (2026) proposed three types of problem chains targeting different reasoning objectives. Analogical question chains, centered on “knowledge commonality,” construct progressive question sequences around similar attributes between known and new knowledge, effectively reducing cognitive jumpiness through “buffer-style” questioning. Inductive question chains, centered on “procedural general methods,” guide students from the particular to the general, from specific to universal, distilling common patterns. Deductive question chains, centered on “verifying conclusions,” follow the logical sequence of major premise → minor premise → specific conclusion → verification, guiding students through the complete deductive reasoning process (Sun, 2026). Ding et al. (2021) emphasized that problem-chain design must align with mathematical core literacy objectives through the design path of “determining core concepts based on core literacy goals—setting main questions according to core concepts—laying out serialized sub-questions around main questions.”

For illustrative purposes, consider a Grade 3 lesson on fraction comparison. An analogical problem chain might begin with comparing fractions sharing the same denominator (e.g., 2/5 vs. 4/5), then progress to fractions sharing the same numerator (e.g., 2/5 vs. 2/7), and finally address fractions with different numerators and denominators (e.g., 3/5 vs. 2/7). Each step activates the reasoning established in the preceding step, creating a cognitive buffer zone that bridges known and unknown knowledge. This scaffolding enables students to construct the general comparison rule inductively rather than receiving it algorithmically, thereby developing both reasoning awareness and strategic flexibility (Tang et al., 2018; Ding et al., 2021).

Cai and Hwang (2002) found that students who engage with generative problem chains of this type demonstrate significantly higher transfer of mathematical reasoning to novel problem contexts compared with students exposed to single-representation instruction. This finding aligns with Kapur's (2016) theory of productive failure: when students encounter a carefully calibrated gap in the problem chain—a question that is challenging but not beyond reach—the 'desirable difficulty' activates deeper processing and strengthens the development of reasoning schemata. Problem-chain design, therefore, should deliberately include such productive challenge points to maximize reasoning development at each level of the three-level progression.

Pathway Two: Optimizing Question Types to Drive Progressive Reasoning

The second pathway involves optimizing the selection and combination of question types according to reasoning objectives. During the reasoning initiation stage, contextual questions and puzzle-shaped questions should be prioritized to stimulate intrinsic motivation for inquiry. During the reasoning deepening stage, provocative mathematical questions and break-down-hard-problems questions serve as primary tools—provocative questions propel students to break through fixed thinking patterns, while break-down-hard-problems questions decompose complex tasks into manageable cognitive steps (Mahmud & Mohd Drus, 2023). Supartini et al. (2024) further indicated that immediate feedback and diverse question formats are key factors in enhancing students' engagement in reasoning. During the reasoning reflection and consolidation stage, clarifying questions and mistake-explaining questions guide students to reflect upon and revise their own reasoning processes at the meta-cognitive level.

Stylianides et al. (2017) observed that the transition from informal exploration to formal argumentation is one of the most challenging pedagogical moves in mathematics instruction. Within the second pathway, this transition is facilitated by sequencing question types from contextual and puzzle-shaped questions (which activate informal reasoning) through provocative and break-down-hard-problems questions (which build toward formal argumentation), and finally to mistake-explaining and clarifying questions (which consolidate and formalize the reasoning chain). This deliberate sequencing transforms what might otherwise be a series of disconnected classroom exchanges into a coherent arc of reasoning development (Stylianides et al., 2017).

Pathway Three: Organizing Reasoning Discourse Through Classroom Discussion

The third pathway involves organizing reasoning discourse to extend the depth of reasoning through classroom discussion. Drawing on the questioning moves model proposed by Livy et al. (2025), teachers should pre-plan question chains based on teaching objectives, clarifying the intended function (facilitating, eliciting, responding, or extending) of each questioning move. Student work samples serve as vital vehicles for reasoning dialogue: displaying student work at varying performance levels and designing comparative questions around sample differences enables students to compare reasoning strategies, identify errors, and construct shared mathematical meanings. This aligns with the core concepts of “knowledge connection” and “method connection” in problem-chain teaching: by displaying and comparing different students' reasoning paths, the entire class is guided to seek commonalities amid differences, deepening their understanding of the nature of mathematical reasoning.

Reznitskaya and Wilkinson (2017) argued that the highest form of classroom reasoning discourse—collaborative argumentation—emerges when students engage in genuine dialogue to evaluate competing arguments rather than merely responding to teacher-initiated questions. Within the problem-chain framework, this collaborative dimension can be fostered by designing divergent sub-questions that explicitly invite students to propose, compare, and adjudicate among multiple reasoning approaches. When teachers deploy extending questioning moves to bring diverse student reasoning paths into class-wide dialogue, students function as active co-constructors of mathematical knowledge rather than passive recipients of procedural instructions. This shift from teacher-directed to student-driven reasoning discourse represents the culmination of the three-pathway framework (Livy et al., 2025; Howe et al., 2019).

Discussions

The proposed framework integrates and extends prior research in several important ways. First, compared with Lannin et al. (2011), who focused on teacher knowledge for reasoning instruction, this framework operationalizes reasoning cultivation through concrete classroom strategies, providing more actionable guidance for practitioners. Second, compared with Mahmud and Mohd Drus (2023), whose six-question taxonomy describes questioning at the micro-level, this framework situates questioning within the broader architecture of problem-chain design and reasoning discourse organization, providing a more systemic approach. Third, compared with Livy et al. (2025), who examined questioning moves in Year 1 classrooms, this framework demonstrates applicability across primary and secondary levels, supported by the three-level reasoning progression model. Fourth, compared with Tang et al. (2018), who documented differences between expert and novice teachers' problem-chain use, this framework articulates the specific design principles that distinguish method-connection-oriented expert practice from knowledge-connection-oriented novice practice. Fifth, consistent with Howe et al. (2019), this framework emphasizes the quality of interactive dialogue as a mediating variable between teacher questioning and student reasoning development, highlighting the importance of teacher responsiveness to students' reasoning states. Sixth, aligned with Mueller et al. (2014), this framework recognizes teachers as central agents in promoting student reasoning, but goes further by providing concrete instructional tools (problem chains, questioning types, questioning moves) that support teacher development.

Seventh, compared with Reznitskaya and Wilkinson (2017), who theorized collaborative argumentation as the apex of classroom reasoning discourse, this framework operationalizes their construct by specifying how problem-chain structures and questioning moves can scaffold collaborative reasoning at each of the three progressive levels identified by Lannin et al. (2011). This translation from argumentation theory to structured instructional practice bridges a longstanding gap between theoretical ideals of mathematical discourse and the practical demands faced by teachers in diverse classroom settings.

Eighth, compared with Cai and Hwang (2002), who examined generative thinking in problem posing from a cross-cultural perspective, this framework extends their cross-cultural lens by providing a structured mechanism—the three-type problem chain—through which generative thinking can be systematically cultivated within Chinese and international mathematics classrooms. Finally, building on Wood's (1998) distinction between funneling and focusing communication patterns and Brodie's (2010) analysis of teacher questioning in secondary classrooms, this framework provides specific strategies for transitioning from funneling questioning (which constrains reasoning) to focusing questioning (which expands it), offering teachers a concrete pathway for improving the quality of their classroom discourse.

CONCLUSION

This study has constructed a comprehensive framework for cultivating mathematical reasoning ability through classroom questioning grounded in problem-chain teaching theory. The framework addresses the research gap identified in the introduction by integrating domestic problem-chain teaching theory with international classroom questioning research, providing a cross-cultural pathway for reasoning cultivation. The central research question—how problem-chain teaching can effectively support mathematical reasoning development—is answered through three complementary pathways: differentiated problem-chain design (analogical, inductive, deductive), optimized oral questioning types (contextual, provocative, clarifying, etc.), and organized reasoning discourse (questioning moves, student work samples).

The framework offers several key contributions. At the theoretical level, it synthesizes Eastern and Western research traditions to provide a comprehensive model for reasoning cultivation.

At the practical level, all three pathways possess strong operability: teachers can immediately apply the analogical-inductive-deductive problem-chain typology, the six-question taxonomy, and the four questioning moves framework to design and implement reasoning-focused instruction. The illustrative lesson design on fraction comparison demonstrates the framework's applicability to authentic classroom contexts.

However, several limitations must be acknowledged. First, the proposed framework relies heavily on teachers' proficiency in designing and adapting problem chains; novice teachers may struggle to balance pre-planned question sequences with real-time responsiveness to students' reasoning states. Future research could develop scaffolding tools such as question-chain templates or professional development modules specifically for early-career teachers. Second, the framework has not yet been empirically tested across different grade bands; the cognitive demands of analogical, inductive, and deductive chains likely vary between lower primary and upper secondary levels. Longitudinal or cross-sectional studies are needed to calibrate the framework for each stage. Third, large class sizes may hinder the organization of reasoning discourse and the effective use of student work samples. Future research could explore hybrid models combining whole-class questioning with small-group reasoning circles. Fourth, all cited studies are from Eastern and Western contexts but do not include comparative analyses; cross-cultural implementation studies would help determine whether the problem-chain approach requires adaptation in different educational cultures. Finally, the cultivation of mathematical reasoning ability is a long-term process, and classroom questioning constitutes only one of its critical components—it must be organically coordinated with factors such as mathematical task design, classroom organizational structures, and formative assessment systems.

ACKNOWLEDGMENT

This work was supported by the School-level Research Project of Guangxi Normal University under Grant XYCS2026128: Research on Multiple Paths and Practical Practice of Generative AI Empowering the Improvement of Core Literacy in High School Mathematical Modeling.

REFERENCES

Ding, F. J., Zhang, W. Z., & Tang, H. J. (2021). Problem-chain teaching design oriented toward mathematical core literacy. Educational Research, (9), 62–66.

Lannin, J., Ellis, A. B., & Elliott, R. (2011). Developing essential understanding of mathematical reasoning for teaching mathematics in grades pre-K-8. National Council of Teachers of Mathematics.

Livy, S., Bobis, J., Downton, A., Russo, J., & Sullivan, P. (2025). Supporting mathematical reasoning: Exploring questioning moves in a Year 1 classroom using student work samples. Mathematics Education Research Journal. https://doi.org/10.1007/s13394-025-00527-2

Mahmud, M. S., & Mohd Drus, N. F. (2023). The use of oral questioning to improve students' reasoning skills in primary school mathematics learning. Frontiers in Education, 8, 1126816. https://doi.org/10.3389/feduc.2023.1126816

Mueller, M., Yankelewitz, D., & Maher, C. (2014). Teachers promoting student mathematical reasoning. Investigations in Mathematics Learning, 7(2), 1–20.

Sun, Y. Z. (2026). Strategies for designing problem chains to cultivate primary school students' awareness of mathematical reasoning. Jiangsu Education Research, (1), 112–117.

Supartini, R., Sari, N. M., Supianti, I. I., & Septian, G. D. (2024). MathQuest: A PowerPoint-based problem-based learning game for mathematical problem-solving. Journal of Advanced Sciences and Mathematics Education, 4(2), 139–152.

Tang, H. J., Tan, H., Xu, Y. G., & Zhang, W. Z. (2018). Research on effective mathematics teaching based on problem chains in middle school: Insights from a lesson study. Journal of Mathematics Education, 27(3), 30–34.

Ellis, A., Ozur, H. B., & Nokes, C. E. (2019). Exploring the role of reasoning and proving in mathematics teaching and learning. International Journal of Research in Education, 42(2), 105–124.

Howe, C., Hennessy, S., Mercer, N., Vrikki, M., & Wheatley, L. (2019). The role of interactive dialogue in students' learning of mathematical reasoning. Learning and Instruction, 60, 101–112. https://doi.org/10.1016/j.learninstruc.2018.11.002

Mullis, I. V. S., Martin, M. O., & von Davier, M. (Eds.). (2021). TIMSS 2023 Assessment Frameworks. IEA. https://timssandpirls.bc.edu/timss2023/frameworks/pdf/T23_Frameworks.pdf

OECD. (2023). PISA 2022 Assessment and Analytical Framework. OECD Publishing. https://doi.org/10.1787/dfe0bf9c-en

Orr, S., & Bieda, K. (2023). Learning to elicit student thinking: The role of planning to support academically rigorous questioning sequences during instruction. Journal of Mathematics Teacher Education. https://doi.org/10.1007/s10857-023-09603-5

Reznitskaya, A., & Wilkinson, I. A. G. (2017). The most reasonable answer: Helping students build better arguments together. Harvard Education Press.

Zhu, Z., & Zhang, L. (2024). TIMSS 2023 mathematics assessment: Features, framework content, and implications. Journal of Mathematics Education, (3), 77–81.

Stylianides, A. J., Bieda, K. N., & Morselli, F. (2017). Proof and argumentation in mathematics education research. In J. Cai (Ed.), Compendium for Research in Mathematics Education (pp. 315–351). National Council of Teachers of Mathematics.

Cai, J., & Hwang, S. (2002). Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing. Journal of Mathematical Behavior, 21(4), 401–421. https://doi.org/10.1016/S0732-3123(02)00142-6

Brodie, K. (2010). Teaching mathematical reasoning in secondary school classrooms. Springer. https://doi.org/10.1007/978-0-387-09742-8

Kapur, M. (2016). Examining productive failure, productive success, unproductive failure, and unproductive success in learning. Educational Psychologist, 51(2), 289–299. https://doi.org/10.1080/00461520.2016.1155457

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), Language and Communication in the Mathematics Classroom (pp. 167–178). National Council of Teachers of Mathematics.