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EXPLORING STUDENT'S MATHEMATICAL REPRESENTATION ABILITY IN DEEP LEARNING-INTEGRATED PROBLEM-BASED LEARNING ON CIRCLE EQUATIONS

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ABSTRACT

Mathematical representation ability is a fundamental competency for senior high school students. This study aims to describe students' mathematical representation abilities in deep learning integrated with the Problem-Based Learning model. A descriptive qualitative approach was employed, involving 36 Grade XI students at SMAN 4 Malang. Data were collected through written tests based on mathematical representation ability indicators and analyzed across representation: verbal, pictorial, and symbolic. The results reveal uneven abilities across representation types: symbolic representation was most consistently fulfilled, though with varied strategies; pictorial representation was spatially constructed but incomplete in metric annotation; verbal representation exhibited the greatest variation, with only high-ability students capable of articulating contextual geometric conclusions. These findings confirm that symbolic proficiency does not guarantee comprehensive representational competence. This study contributes an empirical profile of students' mathematical representation in a PBL context, serving as a reference for teachers in designing balanced instruction across representation types.

Kemampuan representasi matematis merupakan kompetensi fundamental bagi siswa SMA. Penelitian bertujuan mendeskripsikan kemampuan representasi matematis siswa dalam pembelajaran mendalam terintegrasi model Problem Based Learning. Pendekatan kualitatif deskriptif digunakan, melibatkan 36 siswa kelas XI SMAN 4 Malang. Data dikumpulkan melalui tes tertulis berdasarkan indikator kemampuan representasi matematis, dan dianalisis pada representasi: verbal, piktorial, dan simbolik. Hasil menunjukkan kemampuan tidak merata: representasi simbolik paling terpenuhi meski strateginya bervariasi; representasi piktorial terkonstruksi spasial namun tidak lengkap dalam anotasi metrik; representasi verbal paling bervariasi, hanya siswa berkemampuan tinggi mampu mengartikulasikan simpulan geometris kontekstual. Temuan ini menegaskan bahwa kemampuan simbolik tidak menjamin kompetensi representasi menyeluruh. Penelitian berkontribusi memberikan profil empiris representasi matematis siswa pada konteks PBL, sebagai acuan guru merancang pembelajaran yang seimbang antar jenis representasi.

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INTRODUCTION

Mathematics learning in the 21st century demands more than mere procedural mastery and formula memorization. The persistent dominance of procedural, outcome-oriented mathematics instruction has led to a chronic weakness in students' conceptual understanding, underscoring the urgent need for learning approaches that cultivate deep, reflective, and meaningful thinking (Kholid et al., 2025). In the context of 21st-century education, deep mathematical thinking is key to developing higher-order cognitive abilities; however, current mathematics learning predominantly focuses on procedural skills rather than conceptual understanding and reflection, often hindering the optimal development of students' deep mathematical thinking (Orhani, 2024).

One competency regarded as capable of addressing this demand is mathematical representation the ability to express mathematical ideas or concepts through various forms such as symbols, graphs, tables, and verbal language. Mathematical representation is a fundamental competency that students must develop and master, as it supports conceptual understanding, problem-solving, mathematical communication, mathematical connection, and the application of mathematical ideas through modeling (Marliani & Puspitasari, 2022). Despite its central role in fostering deep mathematical thinking, however, mathematical representation remains one of the competencies in which Indonesian students consistently underperform.

Research consistently demonstrates that Indonesian students' mathematical representation ability remains considerably inadequate. Students tend to memorize formulas without genuine conceptual understanding (Handayani & Aini, 2019). Similarly, Panduwinata et al. (2019) found that students struggle to translate problems into alternative forms of representation beyond those with which they are already familiar. Students are also frequently unable to articulate the reasoning behind their answers in coherent terms, reflecting a pronounced weakness in verbal representation (Iffa et al., 2022). These difficulties are particularly evident in circle equation topics, where procedural, lecture-based instruction has been linked to poor performance in standardized examinations (Abreh et al., 2018; Ali, 2022).

Furthermore, students frequently cannot identify what is needed to solve circle-related problems, are unable to formulate an appropriate mathematical model, and commit errors in calculations all of which reflect a fundamental weakness in mathematical representation on this topic (Abidin & Retnawati, 2019). These conditions are further corroborated by the results of PISA 2022, in which Indonesia recorded a mean mathematics score of 366 points substantially below the OECD average of 472 points placing the country 70th out of 81 participating nations (OECD, 2023). More critically, the majority of Indonesian students were only at Level 1 proficiency and none reaching Level 5 or 6, levels that demand mathematical modelling ability directly underpinned by mathematical representation (OECD, 2023).

One instructional approach that holds considerable promise in addressing these challenges is Problem-Based Learning (PBL). PBL is highly suitable for learning that aims to build independent thinking and mathematical creativity, not merely the achievement of standard cognitive scores (Hidayat & Fitrini, 2025). In PBL, students learn by engaging with professionally relevant problems that enhance the application and integration of knowledge, which is assumed to encourage a deep learning approach in which students are intrinsically motivated and genuinely seek to understand what is being studied (Dolmans et al., 2016). Despite its potential, however, existing research has yet to fully explore the relationship between PBL and the depth of students' representational processes. Taken together, these findings suggest that while PBL offers a promising pedagogical foundation for fostering independent and creative mathematical thinking, the extent to which it shapes the depth of students' representational processes remains an open question that warrants further investigation.

Addressing this question requires a closer examination of what deep learning itself entails. Deep learning, in the pedagogical sense, constitutes a qualitatively distinct mode of cognitive engagement that transcends mere information retention and procedural execution (Feriyanto & Anjariyah, 2024; Orhani, 2024). Rooted in the foundational distinction between surface and deep approaches Dolmans et al. (2016) that deep learning is broadly understood as a learning orientation in which students actively seek to understand meaning, critically connect new information with prior knowledge, and transfer conceptual understanding to novel situations. In the Indonesian educational policy context, deep learning encompasses the interconnection between conceptual and procedural knowledge and the capacity to apply conceptual knowledge in new context (Kemendikdasmen, 2025).

This approach is operationalized through three interrelated pillars mindful, meaningful, and joyful learning aimed at developing character and 21st-century skills (Feriyanto & Anjariyah, 2024). Within mathematics education, deep learning principles include conceptual connectivity, intrinsic motivation, and metacognition. These principles serve as foundational elements that, when integrated with mathematical thinking components, give rise to a hierarchical model of deep mathematical thinking. Thus, deep learning as a theoretically grounded framework for cultivating the reflective, meaning-making cognitive processes that underpin genuine mathematical understanding (Orhani, 2024). In short, deep learning offers a theoretically grounded framework for understanding the reflective, meaning-making cognitive processes that underpin genuine mathematical understanding processes that are central to how students construct mathematical representations.

Building on this theoretical foundation, attention must also be given to how mathematical representation itself has been studied. Despite growing scholarly interest in mathematical representation, existing research remains predominantly quantitative in orientation, focusing on the measurement of students' ability levels rather than on exploring the processes and forms of representation students employ during learning (Lutfi & Juandi, 2023; Mainali, 2020). A qualitative approach is considered more suitable for observing how students use mathematical representations in problem-solving, as it enables deeper exploration of representational processes beyond mere ability measurement (Creswell, 2014; Oranga & Matere, 2023). Nevertheless, studies that go beyond ability measurement to examine representational processes in depth remain limited. This indicates that a qualitative, process-oriented lens is needed to capture the depth of mathematical representation in ways that quantitative ability measures alone cannot reveal yet such studies remain limited.

Despite this need, research specifically linking PBL to mathematical representation has rarely adopted such a process-oriented perspective. Studies linking PBL to mathematical representation have largely examined the improvement of representational ability as an outcome of instruction, without probing how students' mathematical representations are constructed throughout the learning process (Sari et al., 2023). Furthermore, research that explicitly connects deep learning with mathematical representation within the context of mathematics education remains scarce Patmaniar et al. (2025) despite the fact that depth of conceptual understanding constitutes the very foundation of meaningful mathematical learning. Collectively, these gaps point to an underexplored intersection between PBL, deep learning, and the process by which mathematical representation is constructed an intersection that the present study seeks to address.

It is precisely this intersection that motivates the integration of deep learning into PBL in the present study. The integration of deep learning into Problem-Based Learning (PBL) is necessitated by a fundamental limitation of PBL alone. Although PBL is assumed to encourage intrinsic motivation and genuine understanding (Dolmans et al., 2016), empirical evidence shows it enhances deep learning with only a small positive average effect size, with several studies showing no effect (Fitriyah et al., 2022; Paloloang et al., 2020; Suparman et al., 2021) or even a decrease (Dolmans et al., 2016; Kho et al., 2024). Dolmans et al. (2016) indicating that deep learning must be deliberately cultivated within the learning design rather than assumed as its natural by-product. Research confirms that integrating deep learning within PBL significantly improves students' conceptual understanding and problem-solving ability in mathematics and further strengthens reflective engagement crucial for higher-order cognitive development (Bariroh, 2025; Sofa et al., 2025).

In the context of mathematical representation specifically, without the mindful, meaningful, and joyful dimensions of deep learning, students may engage with PBL tasks at a purely procedural surface level manipulating symbols without constructing the conceptual connectivity that enables flexible representation across symbolic, graphical, verbal, and visual modes. Since deep learning fundamentally emphasizes conceptual understanding and the critical application of knowledge (Mutmainnah et al., 2025). The present study positions it as an explicit pedagogical orientation integrated within PBL to shape how students construct mathematical representations on circle equations.

It is this research gap that necessitates a study integrating deep learning with the Problem-Based Learning model, with a specific focus on the processes and forms of mathematical representation that emerge during instruction. Students are expected to develop mathematical representation skills and apply them flexibly in problem-solving situations (Maryati & Monica, 2021; Ramadhan & Aini, 2021; Suwanti & Maryati, 2021), using various representations such as symbols, diagrams, graphs, images, and other visual aids. Therefore, a quality mathematics learning process needs to provide opportunities for students to explore and enhance these representational abilities (Inayah & Dasari, 2023; Khoerunnisa & Maryati, 2022).

Mathematical representation is fundamentally tied to students' capacity to articulate their conceptual understanding of mathematics in multiple ways; weaknesses in this area, therefore, must be addressed through purposeful and well-designed instructional approaches. A qualitative approach is particularly appropriate for this purpose, as it can understand phenomena from the participants' perspective and capture experiences and meanings that cannot be measured with numbers, thereby enabling a deeper portrait of students' thinking that goes beyond scores and rankings (Oranga & Matere, 2023).

It is hoped that the findings of this study will make a meaningful contribution to the design of mathematics instruction that promotes deep conceptual understanding, in alignment with the demands of the Merdeka Curriculum and 21st-century education (Kholid et al., 2025). Based on the foregoing, this study aims to describe students' mathematical representation in deep learning integrated with the Problem-Based Learning model on the topic of circle equations.

METHOD

This study employed a qualitative descriptive research design, which aims to describe and interpret phenomena as they naturally occur without manipulating variables. In qualitative research, the methods section requires careful reflection on the researcher's role in the study, drawing from multiple data sources, using specific protocols for data recording, analyzing information through multiple steps, and documenting approaches for ensuring data validity (Creswell, 2014). This approach was selected because the primary objective of the study is to describe in depth the forms and processes of students' mathematical representation during deep learning integrated with the Problem-Based Learning (PBL) model on the topic of circle equations.

This study was conducted at SMAN 4 Malang, East Java, Indonesia, during the academic year 2025/2026. The research was carried out in XI Mechanical and Digital, during the implementation of PBL-based deep learning on the topic of circle equations. A total of 36 students participated in this study. Participants were selected using a purposive sampling technique, in which subjects are chosen not randomly but based on specific criteria relevant to the research objectives. Purposive sampling also referred to as judgment sampling or theoretical sampling involves the selection of cases with a specific goal or purpose in mind, ensuring that the selected participants are able to provide rich and relevant information in relation to the phenomenon under study (Miles et al., 2014). In this study, the selection criteria included: (1) active participation in PBL-based learning activities, and (2) the ability to produce written work that reflects diverse forms of mathematical representation across verbal, pictorial, and symbolic categories. The sequential stages of this research are depicted in the flowchart shown in Figure 1.

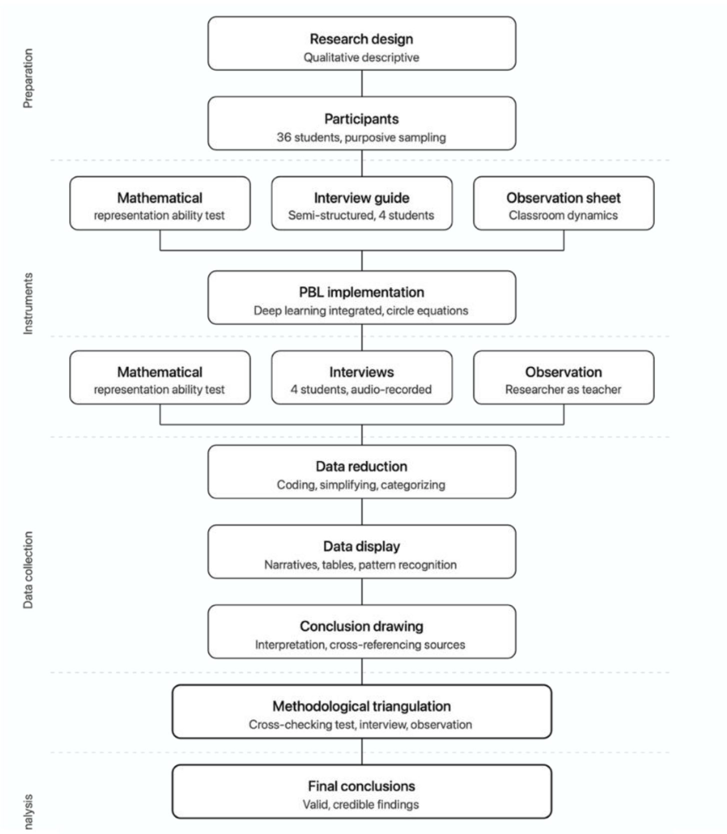

Figure 1. Flowchart of The Reseacrh Procedure

The primary instrument in this study was the researcher, who functioned as the key instrument of data collection, observation, and analysis. This is consistent with the epistemological nature of qualitative inquiry, which refers to the foundational assumption that knowledge is constructed through detached measurement (Creswell, 2014). Supporting instruments were developed to guide data collection systematically. First, a mathematical representation ability test instrument was constructed, consisting of contextual problem-based tasks specifically designed to elicit students' verbal, pictorial, and symbolic representations on the topic of circle equations. Second, a interview guide was developed, in which questions are partially predetermined but allow flexibility for follow-up based on students' responses.

The interview was conducted with four selected students and covered three main indicator areas: (1) students' mathematical representation ability, encompassing verbal, pictorial, and symbolic forms; (2) students' strategy selection in solving problems; and (3) students' explanations of the answers they had written in the test. Third, an observation sheet was used to systematically record classroom interactions, student engagement, and learning dynamics throughout PBL implementation.The observations were conducted directly by the researcher, who also served as the classroom teacher during the study.

The classification of mathematical representation adopted in this study follows the framework proposed by Villegas et al. (2009) who categorize external mathematical representation into three types: (1) verbal representation, which is fundamentally expressed in writing or speech; (2) pictorial representation, including images, diagrams, and graphs; and (3) symbolic representation, consisting of numbers, operational signs, algebraic symbols, and mathematical relations. Data were collected through four methods: a mathematical representations ability test, semi-structured interviews, classroom observation, and documentation. The written test was designed to elicit students' mathematical representations on circle equations. Interviews were conducted to explore students' reasoning and thinking processes, audio-recorded, and transcribed verbatim prior to analysis (Ruslin et al., 2022). Observation and documentation served to capture naturally occurring representational behaviors throughout learning. To operationalize this framework in the context of circle equations, indicators were developed for each representation type as presented in Table 1.

Table 1. Indicators Mathematical Representation

Representation Type	Indicators
Verbal	Students are able to explain the position of a point relative to a circle (inside, on, or outside) in written or oral form.
	Students are able to describe the steps for determining the position of a point relative to a circle.
	Students are able to interpret known and asked information from word problems related to the position of a point relative to a circle.
Pictorial	Students are able to draw a circle along with a point on a coordinate plane.
	Students are able to illustrate the position of a point relative to a circle through a diagram or graph.
	Students are able to accurately label the center, radius, and point location in the figure.
Symbolic	Students are able to correctly write the equation of a circle.
	Students are able to apply the distance formula from a point to the center of a circle to determine the position of the point.

	Students are able to perform mathematical operations and calculations to determine whether a point lies inside, on, or outside the circle.
	Students are able to use symbols or mathematical models to solve problems involving the position of a point relative to a circle.

Data were analyzed using the interactive model of Miles et al. (2014), encompassing three iterative stages: data reduction, data display, and conclusion drawing and verification. In the data reduction stage, raw data from the written test, interviews, and observations were coded and simplified based on three types representation indicators: verbal, pictorial, and symbolic. In the data display stage, the reduced data were organized into descriptive narratives and categorized tables to facilitate pattern recognition. In the conclusion drawing and verification stage, interpretations were drawn and verified through cross-referencing multiple data sources.

To ensure credibility, methodological triangulation was conducted by cross-checking findings across three sources. Students' written test responses were analyzed to identify their representation, then compared against interview data from four purposively selected students who elaborated on their strategy choices and solution explanations. These findings were further corroborated with observational data recorded by the researcher, who simultaneously served as the classroom teacher, to minimize interpretive bias and strengthen the validity of conclusions (Denzin, 2009; Patton, 2002).

RESULT AND DISCUSSION

Result

The mathematical problem administered to the students consisted of a mathematical representation ability test. The researcher employed one contextual problem containing three sub-questions, in which each sub-question was designed to measure a specific form of representation according to the established indicators. Each sub-question represented one indicator of representational ability. The forms of representation assessed included pictorial representation, symbolic representation, and verbal representation. In solving the problem, each student demonstrated different levels of understanding as well as distinct approaches in representing the given mathematical problem. The given problem can be seen in **Figure 2.**

KODE A

3. Pada sebuah panggung, seorang penata lampu menggunakan lampu sorot untuk menyinari area panggung. Sinar yang dihasilkan dari lamu sorot berbentuk lingkaran dengan pusat di (13,4) dan radius sinarnya 5 meter.
   a. Tentukan persamaan umum lingkaran dari sinar yang dibentuk oleh lampu sorot!
   b. Jika tiga penampil, yakni Adi, Budi, dan Citra berutut-turut ada pada koordinat (11,4), (8,5), dan (17,7), tentukan posisi mereka terhadap lampu sorot!
   c. Buatlah gambar lingkaran yang dihasilkan lampu sorot dan posisi Adi, Budi, dan Citra pada bidang Kartesius!

Figure 2. Contextual Problem Given to The Students

Verbal representation was assessed through three indicators, adapted from the framework proposed by the Villegas et al. (2009): (a) explaining the position of a point relative to a circle in written or oral form, (b) describing the procedural steps for determining point positions, and

(c) interpreting known and required information from the word problem. These indicators align with the classification of verbal representation as written text used to express mathematical ideas.

The four subjects S1, S2, S3, and S4 were selected through purposive sampling based on two criteria: (1) active participation in PBL-based learning activities, and (2) the ability to produce written work reflecting diverse forms of mathematical representation across verbal, pictorial, and symbolic categories. The four subjects demonstrated varying levels of proficiency in expressing the position of points relative to the circle in both written and oral forms. The work results of the four students can be seen in Figure 3, Figure 4, and Figure 5.

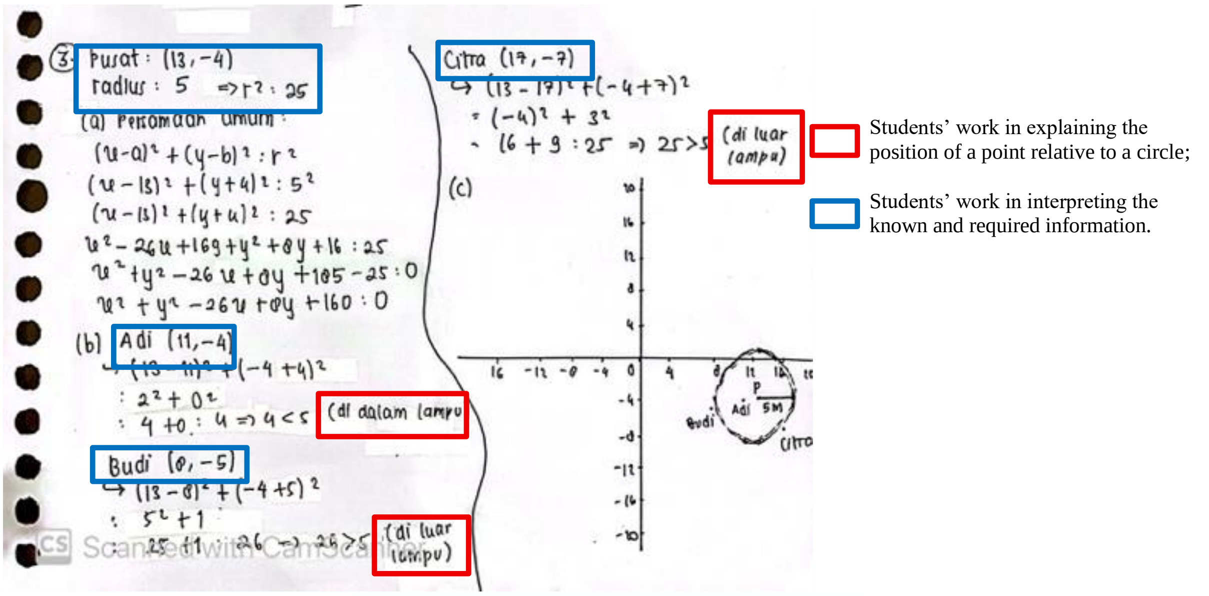

Figure 3. S1's Solution to The Problem

As shown in Figure 3, S1 expressed the conclusions regarding point positions in writing using the phrases *"di dalam lampu"* for Adi's point and *"di luar lampu"* for both Budi's and Citra's points. S1's conclusion was somewhat ambiguous regarding the distinction between inside, outside, and on the lamp. The conclusion should instead have referred to the "light beam" or "spotlight coverage" rather than the lamp itself. This verbal inaccuracy indicates a disconnect between S1's computational result and her ability to interpret its geometric meaning in either written or oral form.

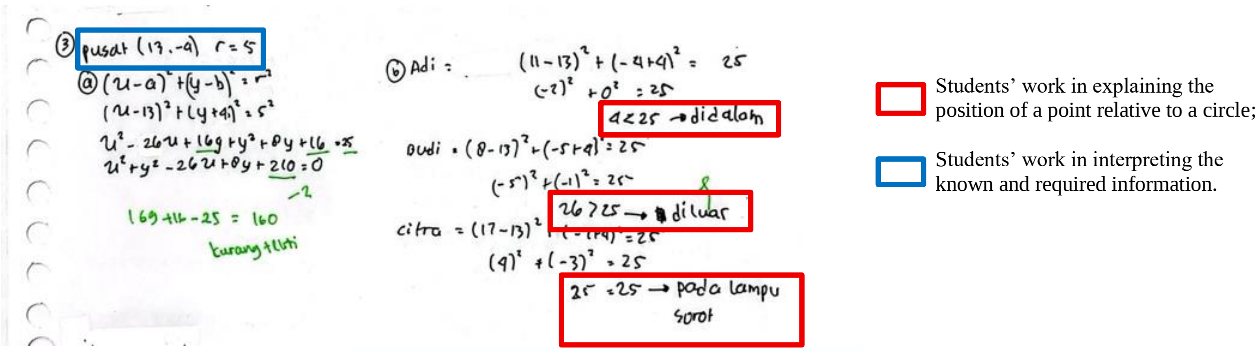

Figure 4. S2's Solution to The Problem

In contrast, S2 demonstrated more complete and accurate verbal ability. As evidenced in Figure 4, S2 wrote correct conclusions for all three points and was further able to articulate them coherently during the interview: *"Kalau Adi berada di dalam jangkauan lampu sorot karena jaraknya itu kurang dari jari-jari lingkaran, kalau yang Budi nanti setelah disubstitusikan hasilnya itu 26, jadi posisinya di luarnya, dan Citra karena dia itu sama dengan jari-jarinya, maka dia itu berada tepat pada lingkaran"* meaning "Adi is within the spotlight range because the distance is less than the radius of the circle, as for Budi, after substitution the result is 26, so his position is outside the circle and Citra, because her distance equals the radius, is located exactly on the circle". These statements demonstrate that S2's verbal representation was coherently grounded in conceptual understanding, not merely procedural execution.

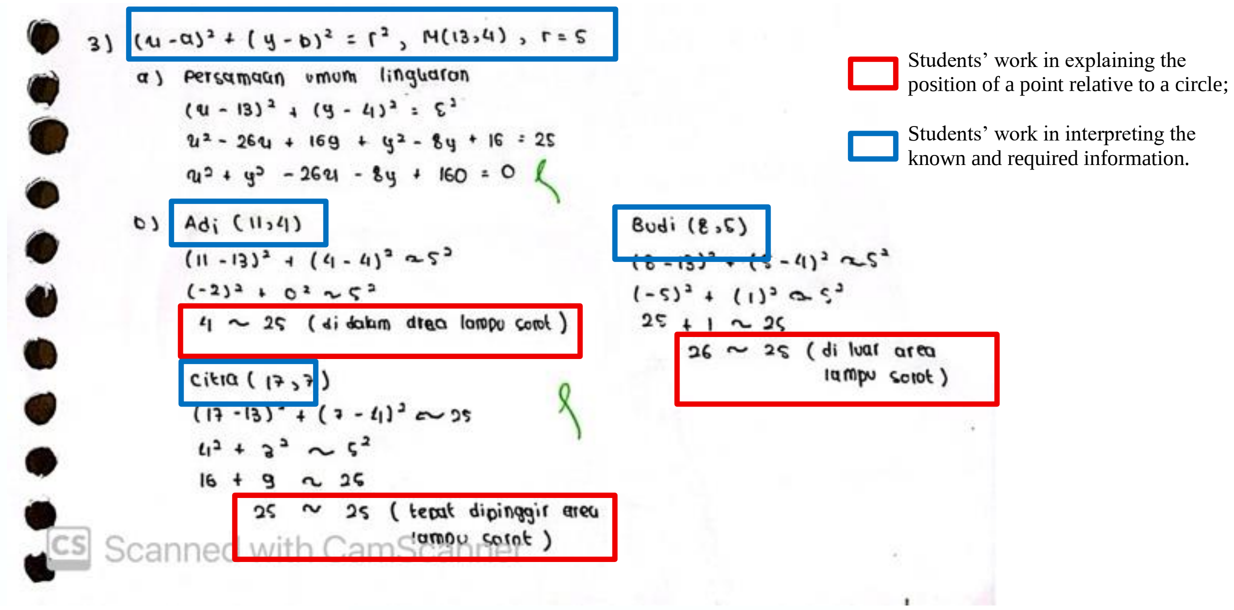

Figure 5. S3's Solution to The Problem

Similarly, As evidenced in Figure 5, S3 correctly explained the positions of all three points during the interview: *"Budi itu menghasilkan 26 yang di mana lebih besar dari 25, yaitu di luar lingkaran. Lalu Citra itu hasilnya 25 yang sama dengan 25, jadi terletak pada lingkaran"* which can be translated as "Budi yields 26, which is greater than 25, meaning he is outside the circle. Then Citra yields 25, which equals 25, so she is located on the circle". Therefore, in this first indicator, S2 and S3 fully fulfilled the indicator, whereas S1 fulfilled it only partially due to verbal misinterpretation in the case of Citra's point. The written work and interview results of S4 were largely consistent with those of S3, indicating a similar pattern of mathematical representation.

The four subjects demonstrated varying levels of verbal ability in terms of completeness and depth of procedural explanation. S1, S2, S3, and S4 fulfilled this indicator well, S2 described the procedure in a structured and conceptually grounded manner, as shown in Figure 4. During the interview, S2 explained: *"Untuk mencari kedudukan titiknya kita tinggal substitusi aja ke persamaan lingkaran. Kalau tadi kan ke (a) sama (b)-nya, kalau sekarang ke (x) sama (y)-nya,"* meaning *"To find the position of the point, we just substitute it into the circle equation. If before it was into (a) and (b), now it's into (x) and (y)."* Furthermore, S2 was able to articulate the general principle underlying point-position determination: *"Jadi antara jarak titiknya itu dibandingkan sama jari-jarinya. Kalau misalnya jaraknya itu kurang dari jari-jari, di dalam.*

Kalau jaraknya itu sama sama jari-jari, pada, dan kalau misalnya lebih, di luar," translate as *"So the distance of the point is compared to the radius. If the distance is less than the radius, it is inside. If the distance equals the radius, it is on the circle. And if it is greater, it is outside."* These verbal explanations indicate that S2 understood the logical structure of the procedure conceptually rather than merely operationally.

S1, S3, and S4 similarly demonstrated a strong ability to translate mathematical procedures into structured and comprehensible language during the interview: *"Nah jadi kita harus menentukan posisi mereka, itu dengan cara kita memasukkan koordinat mereka... itu kita kurangkan dengan pusat (a), yaitu (11) kurang (13)... lalu kita masing-masing kuadratkan... nah lalu kita hitung menjadi 4. Nah 4 ini kan kita harus menentukan letaknya, nah karena lebih kecil dari radius dikuadrat yaitu lebih kecil dari 25, maka terletak di dalam lingkaran,"* meaning *"So we need to determine their positions, by substituting their coordinates... we subtract the center (a), which is (11) minus (13)... then we each square them... and then we calculate to get 4. Now for this 4 we need to determine its position since it is less than the radius squared, which is less than 25, it is located inside the circle."* Therefore, S1, S2, and S3 fulfilled this indicator well.

All four subjects demonstrated relatively similar ability in identifying the key elements of the problem, and this indicator was generally fulfilled by all subjects, though with differing depths of interpretation. As shown in Figure 3, Figure 4, and Figure 5, S1 began solution by writing the known information, namely the center point (13,−4) and radius r = 5, indicating ability to identify the primary elements of the given problem. S3 and S4 similarly identified the known quantities and additionally enumerated all sub-questions to be addressed including the general equation of the circle, the determination of point positions, and the construction of the graph reflecting a comprehensive reading of the problem.

S2 demonstrated the richest depth of interpretation among the three subjects. As shown in Figure 4, S2 not only identified the given information but was also able to relate the contextual terminology of the problem to the corresponding mathematical concept, as reflected in statement: *"Yang diketahui itu ada pusatnya di titik M(13,−4) dan radius sinarnya yaitu 5 meter. Nah, radius-radiusnya ini sama dengan jari-jari,"* meaning *"What is known is that the center is at point M(13,−4) and the beam radius is 5 meters. So, these radii are the same as the radius of the circle."* This ability to bridge real-world context with mathematical terminology reflects a deeper level of verbal representational understanding. Therefore, all four subjects generally fulfilled this indicator, with S2 demonstrating the greatest conceptual depth in contextual interpretation.

Pictorial representation was assessed through three indicators, adapted from the framework proposed by the Villegas et al. (2009) : (a) drawing a circle along with points on the coordinate plane, (b) illustrating the position of points relative to the circle through a diagram or graph, and (c) accurately labeling the center point, radius, and point locations in the figure. The analysis of pictorial representation focuses primarily on S3, as S3's figure best fulfills the criteria established for this study. While S1, S2, and S4 also produced graphical representations, their figures contained notable inaccuracies including imperfect circular shapes and inconsistent distance ratios between the Cartesian axes which rendered them less suitable for in-depth pictorial analysis. S3's figure, by contrast, demonstrated the greatest degree of geometric accuracy and completeness, making it the most appropriate basis for evaluating pictorial representational ability in this section. S3's graphical solution to item 3c is presented in Figure 6.

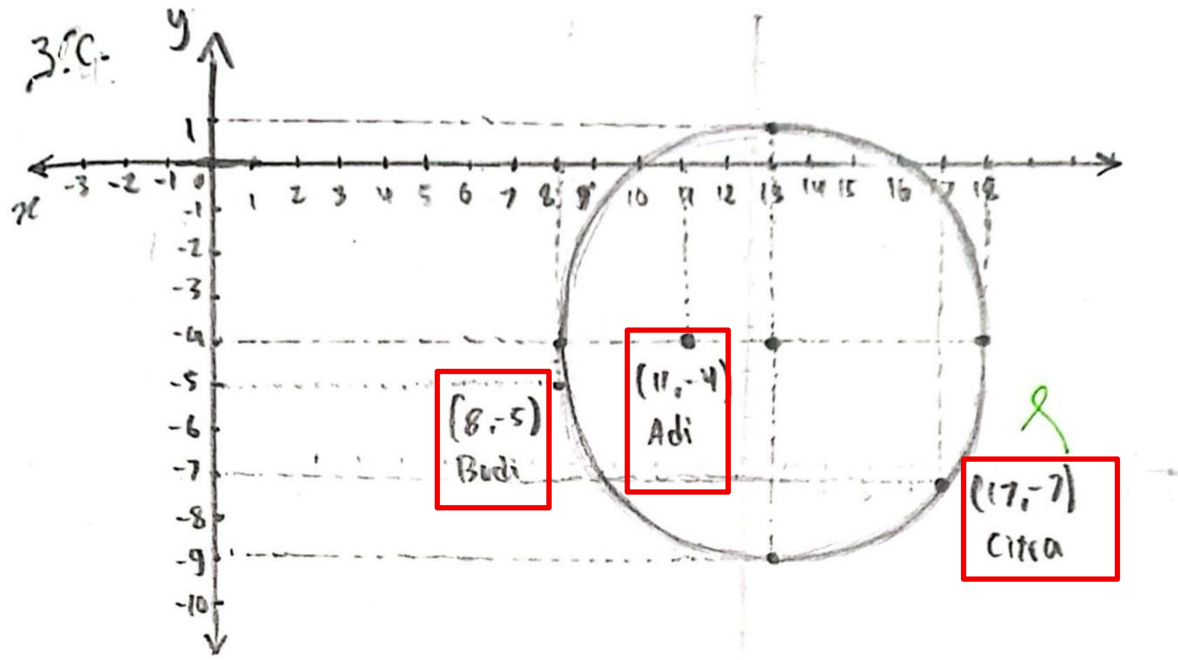

Figure 6. S3's Solution to Item 3c

S3 demonstrated good proficiency in constructing the geometric figure, as shown in Figure 6. S3 built a Cartesian coordinate system with explicitly labeled x- and y-axes, complemented by an adequate numerical scale to represent the positions of all relevant points. Before sketching the circular curve, S3 first determined four cardinal boundary points based on the radius, as described during the interview: *"Pertama-tama saya menggambar grafik sumbu Kartesius, lalu saya tentukan titik pusatnya yaitu (13,-4), lalu saya gambar titik di situ. Lalu saya gambar juga titik di... karena itu jari-jarinya 5 satuan, jadi saya gambar titik ke atas 5 satuan yaitu di (13, 1), lalu 5 satuan ke kanan (18,-4), dan 5 satuan ke bawah (13,-9), dan 5 satuan ke kiri (8, -4). Nah setelah saya tentukan itu semua, lalu saya gambar menggunakan jangka,"* meaning *"First, I drew the Cartesian coordinate axes, then I determined the center point at (13,−4) and marked it there. Then I also drew points at... since the radius is 5 units, I drew a point 5 units upward at (13, 1), then 5 units to the right at (18,−4), then 5 units downward at (13,−9), and 5 units to the left at (8,−4). After determining all of those, I then drew the circle using a compass."* This approach reflects S3's understanding of the geometric properties of a circle pictorially, as he derived the boundary reference points from the radius before completing the sketch. The deliberate use of a compass further demonstrates S3's awareness of the importance of precision in visual mathematical representations. Therefore, the first indicator of pictorial representation was satisfactorily fulfilled by S3.

S3 demonstrated adequate proficiency in illustrating the positions of the three points relative to the circle, as presented in Figure 6. The points Adi (11, −4), Budi (8, −5), and Citra (17, −7) were plotted on the coordinate plane in positions fully consistent with the symbolic analysis: Adi's point was illustrated inside the circle, Budi's point outside the circle, and Citra's point exactly on the circumference. This visual consistency was confirmed during the interview: *"Ternyata titik Adi itu kalau misalnya di (11, -4) itu berada di dalam lingkaran, pun sama Budi kalau digambarkan ternyata memang di luar, sedangkan Citra ini tepat pada lingkarannya,"* meaning *"It turns out that point Adi at (11, −4) is located inside the circle, and likewise Budi, when plotted, is indeed outside, whereas Citra is exactly on the circle."* The correspondence between the visual positions of the three points and the symbolic calculations demonstrates that S3's pictorial representation functioned as a coherent visual confirmation of his mathematical analysis. Therefore, the second indicator of pictorial representation was satisfactorily fulfilled by S3.

S3 demonstrated fairly good proficiency in labeling, though with a notable shortcoming, as can be observed in Figure 6. S3 correctly labeled the x- and y-axes, the center point, and the coordinates and names of all three points Adi (11,−4), Budi (8,−5), and Citra (17,−7). However, the radius distance was not explicitly annotated in the figure. Although S3 clearly understood and utilized the radius value throughout the drawing process, the absence of an explicit numeric radius annotation meant that the figure was not yet fully self-contained as a complete and informative mathematical diagram. As reflected in statement: *"Tidak saya tulis, tapi saya tahu jari-jarinya 5. Saya pikir sudah cukup dengan menunjukkan lingkarannya,"* meaning *"I didn't write it, but I already knew the radius was around 5. I thought it was enough to show the circle."* Therefore, the third indicator was only partially fulfilled: the labeling of point coordinates and coordinate axes was adequate, whereas the annotation of the radius as a metric property of the circle still required improvement.

Symbolic representation was assessed through four indicators, adapted from the framework proposed by the Villegas et al. (2009): (a) correctly writing the equation of a circle, (b) applying the distance formula from a point to the center of the circle to determine point positions, (c) performing accurate mathematical operations and calculations, and (d) using appropriate mathematical symbols and notation in problem-solving.

Both S1 and S4 fulfilled this indicator satisfactorily, though through distinctly different symbolic strategies, as evidenced in Figures 7 and Figure 8 respectively. It should be noted that S2 and S3 produced responses both in their written work and interview answers that were essentially identical to those of S1; therefore, S1 is presented here as a representative of this group, and separate discussion of S2 and S3 is omitted to avoid redundancy.

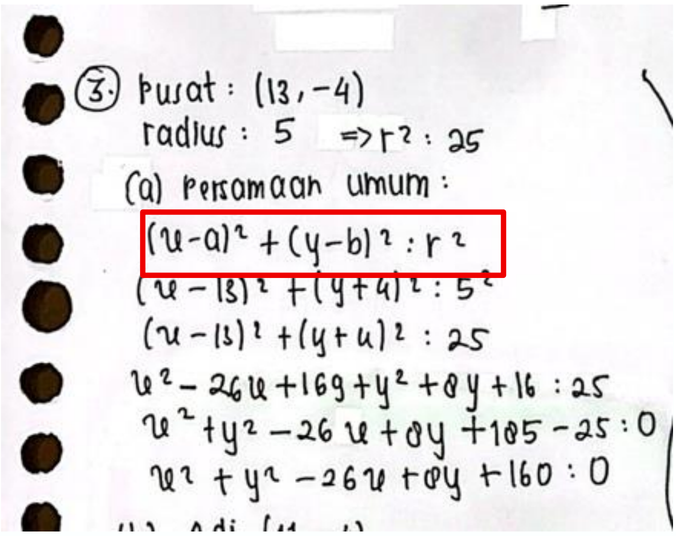

Figure 7. Symbolic Representation in S1's Problem Solving

As shown in Figure 7, S1 began by identifying all given information before proceeding with the formula application. This initial step was described during the interview*: "Yang pertama pasti saya menuliskan apa yang diketahui atau informasi-informasi apa saja yang ada pada soal. Nah di sini kan ada pusatnya (13, -4) dan radiusnya adalah 5,"* meaning *"The first thing I always do is write down what is known, or what information is available in the problem. Here, we have the center at (13, −4) and the radius is 5."* S1 then wrote the standard form of the circle equation and substituted the center M(13,−4) and radius r=5, as she explained: *"Rumus* *persamaan umum jika diketahui pusatnya adalah (a,b) itu rumusnya adalah* $(x-a)^2+(y-b)^2=r^2$*. Nah di sini kita tinggal substitusikan nilai-nilai yang diketahui pada soal, yakni a-nya adalah 13 dan b-nya adalah -4,"* meaning *"The general formula when the center is given as (a, b) is* $(x-a)^2+(y-b)^2=r^2$*. Here, we simply substitute the known values from the problem, where a is 13 and b is −4."* This statement indicates that S1 did not merely apply the procedure mechanically, but also understood the function of each variable within the symbolic structure of the circle equation. Through systematic algebraic expansion, S1 correctly arrived at the general form, as she confirmed: *"Nah di situ karena yang disuruh adalah persamaan umum lingkarannya, jadinya tinggal dijabarkan saja, jadi hasilnya adalah* $x^2+y^2-26x+8y+160=0$*,"* meaning *"Since what was required was the general form of the circle equation, I simply expanded it, giving the result* $x^2+y^2-26x+8y+160=0$*."*

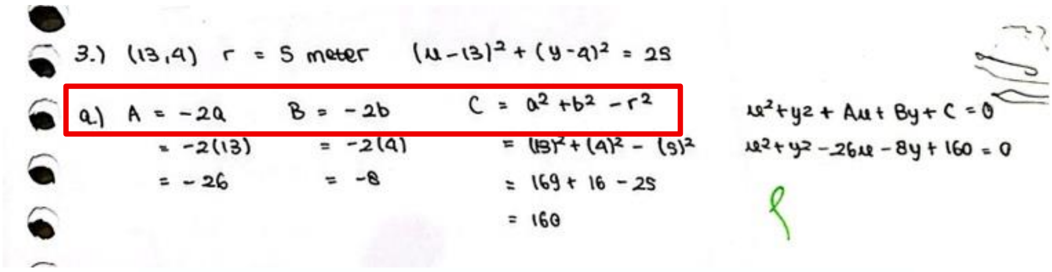

Figure 8. Symbolic Representation in S4's Problem Solving

As shown in Figure 8, S4 employed a more analytical-deductive approach by directly computing the coefficients A, B, and C of the general form using the derived formulas, and ^{2}, bypassing the stage of algebraic expansion entirely. This was reflected in statement: "Oleh karena titik pusat di (13, 4) maka nilai a = 13 dan b = 4, berjari-jari 5, maka r = 5. Dari situ saya bisa substitusikan nilai a, b, dan r untuk mencari besaran A, B, dan C untuk membentuk persamaan lingkaran," meaning *"Since the center is at (13, 4), the value of a = 13 and b = 4, with a radius of 5, so r = 5. From there, I could substitute the values of a, b, and r to find the coefficients A, B, and C in order to form the circle equation."* Despite employing a different strategy from S1, S4 arrived at the same correct general equation $x^2 + y^2 - 26x + 8y + 160 = 0$. This finding suggests that both subjects possessed adequate symbolic understanding of circle equations, with S1 demonstrating a step-by-step transformational approach and S4 demonstrating a formula-driven deductive approach.

As shown in Figure 9, S2 demonstrated good and consistent understanding in this indicator. S2 applied the expression to each tested point and compared the result to r^2, using this structural pattern identically for all three points Adi, Budi, and Citra. When asked to articulate the underlying principle, S2 stated: "*Jadi antara jarak titiknya itu dibandingkan sama jari-jarinya. Kalau misalnya jaraknya itu kurang dari jari-jari, di dalam. Kalau jaraknya itu sama sama jari-jari, pada dan kalau misalnya lebih, di luar,*" meaning *"So the distance of the point is compared to the radius. If the distance is less than the radius, the point is inside. If the distance equals the radius, the point is on the circle and if the distance is greater, the point is outside."*. This statement demonstrates that S2 not only applied the formula procedurally but also understood the relational meaning between a point's distance to the center and the radius as the geometric basis for determining point position. Therefore, the second indicator of symbolic representation was satisfactorily fulfilled by S2.

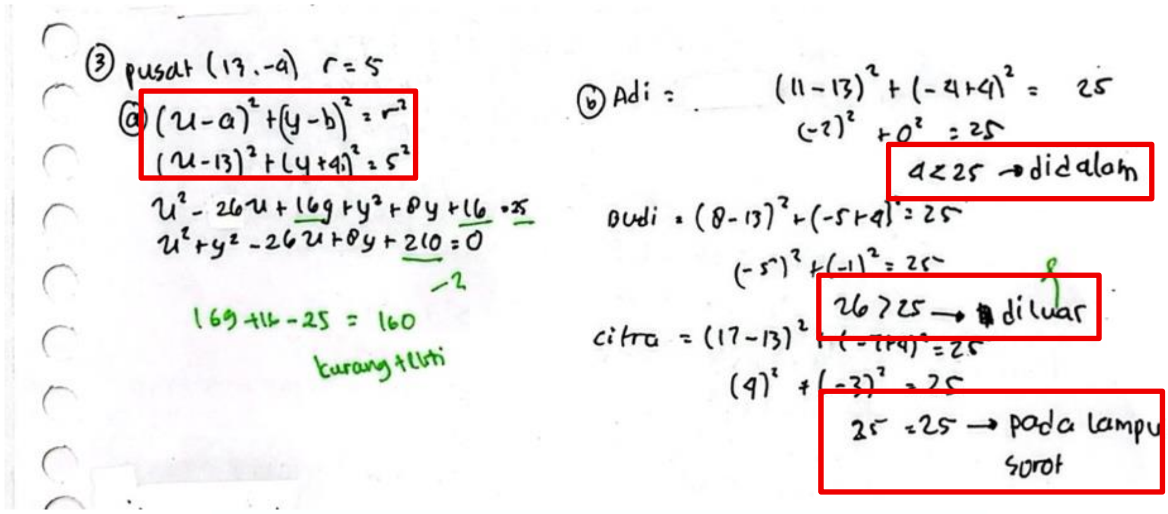

Figure 9. Symbolic Representation in S2's Problem Solving

Figure 9 presents S2's written solution for the third indicator of symbolic representation, in which S2 was required to determine whether each of the three points Adi, Budi, and Citra lies inside, outside, or on the circle. The figure illustrates S2's systematic application of the distance substitution expression to each point and the subsequent comparison of the result against $r^2 = 25$. As visible in the written work, S2 carried out the computations accurately and drew correct conclusions for all three points, demonstrating not only procedural fluency but also a coherent understanding of the geometric relationship between a point's distance to the center and the boundary defined by the radius. This consistency across all three cases confirms that S2's symbolic reasoning in this indicator was both reliable and conceptually grounded.

As evidenced in Figure 9, S2 demonstrated accurate and systematic computational ability for all three tested points. For the point Adi, the substitution yielded a value less than $r^2=25$, indicating that Adi lies inside the circle. For the point Budi, the result of 26 exceeded $r^2=25$, indicating that Budi lies outside the circle. For the point Citra, the substitution yielded exactly 25, equal to $r^2=25$, indicating that Citra lies precisely on the circumference. All computational operations and conclusions were correct and consistent across all three points. This was further reinforced during the interview, in which S2 explained: *"Kalau Adi berada di dalam jangkauan lampu sorot karena jaraknya itu kurang dari jari-jari lingkaran, kalau yang Budi nanti setelah disubstitusikan hasilnya itu 26, jadi posisinya di luarnya* dan *Citra karena dia itu sama dengan jari-jarinya, maka dia itu berada tepat pada lingkaran*" meaning *"Adi is within the range of the spotlight because the distance is less than the radius of the circle, for Budi, after substitution the result is 26, so his position is outside the circle, for Citra, because the result equals the radius, she lies precisely on the circle."* These statements reflect a strong coherence between the symbolic computational processes recorded in Figure 5 and S2's conceptual understanding. Therefore, the third indicator of symbolic representation was satisfactorily fulfilled by S2.

S2 consistently employed relational symbols (<, >, =) as instruments for concluding the position of each point, and used standard notation throughout his quadratic expressions and arithmetic operations. The work results of S2 can be seen in Figure 10.

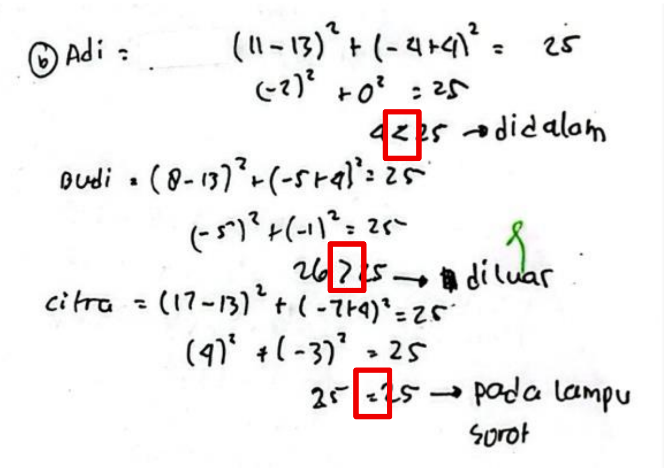

Figure 10. Symbolic Representation in S2's Problem Solving

As illustrated in Figure 10, S2 relied on relational symbols (<, >, =) as the primary operational tool for drawing conclusions at each stage of the solution. After substituting each point's coordinates into the circle equation, S3 compared the resulting value against the squared radius (25) using the appropriate relational symbol: "<" to indicate that Adi's position fell inside the spotlight's range, ">" to indicate that Budi's position fell outside the range, and "=" to indicate that Citra's position lay exactly on the boundary of the spotlight. This consistent and accurate use of relational symbols demonstrates that S2 treated symbolic notation not merely as a computational device, but as a meaningful instrument for translating numerical results into geometric conclusions.

A notable distinction from S1 was S3's consistent use of the equality sign (=), whereas S1 tended to substitute it with a colon (:) when expressing the relationship between the computed value and the squared radius. The corresponding work samples of S1 and S3 are presented in Figures 10 and 11, respectively.

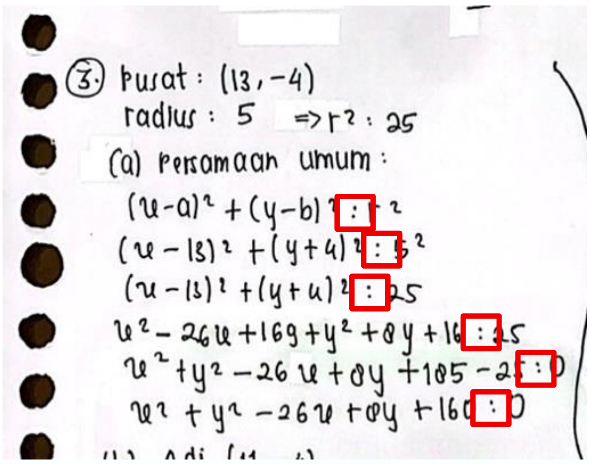

Figure 11. Symbolic Representation in S1's Problem Solving

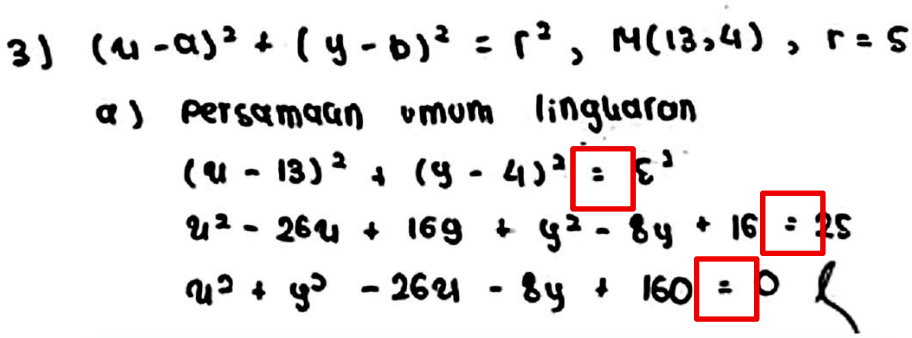

Figure 12. Symbolic Representation in S3's Problem Solving

This substitution suggests that S1 used the colon as a general connector linking a computation to its result, rather than as a precise mathematical symbol denoting equivalence, which points to a less rigorous command of standard symbolic notation. In contrast, S3's consistent and appropriate use of the equality sign reflects a stronger understanding of formal mathematical convention in expressing relational statements.

Discussion

This study examined the mathematical representation abilities of SMA students in solving a contextual problem involving circle equations, with analysis organized around three representational modalities: verbal, pictorial, and symbolic. The central finding is that students demonstrated differentiated and uneven profiles across these three forms, with symbolic representation emerging as the most consistently fulfilled domain, followed by verbal representation, while pictorial representation exhibited the most nuanced incompleteness particularly in the area of diagram annotation and labeling. This hierarchical pattern is broadly consistent with findings in the literature on mathematical representation at the SMA level.

Sanjaya et al (2018), using the Honey and Mumford framework activist, reflector, theorist, and pragmatis found that symbolic representation tended to be better developed across all learning style types, while verbal and visual representation remained merely adequate rather than proficient. A closely parallel finding was reported by Lutfi & Khusna (2021), who studied class XI SMA's mathematical representation abilities on circle intersection material and found that even high-motivation students, while generally competent in symbolic manipulation, failed to fulfill at least one verbal representation indicator. At the theoretical level, this differentiated representational profile corroborates Mainali (2020) that mathematical representation is a multidimensional competency encompassing verbal, graphical, algebraic, and numerical modes each of which requires distinct cognitive resources and should be treated as an independent instructional objective rather than an assumed by product of symbolic fluency.

The analysis of verbal representation reveals a nuanced spectrum of proficiency across the four research subjects. Although all four students succeeded in identifying the key given information from the contextual problem, they differed substantially in their capacity to articulate the geometric meaning of their computational results in accurate and contextually appropriate language. S1's verbal inaccuracy in which she substituted the geometrically precise referent of 'spotlight coverage' with the colloquial term 'the lamp itself' reflects a well-documented challenge at the SMA level: students may demonstrate procedural correctness in symbolic computation yet fail to maintain semantic coherence when verbalizing the contextual and geometric implications of their results. Lutfi & Khusna (2021) specifically noted that students with medium motivation who worked on SMA-level circle material produced symbolically correct answers but struggled at the verbal articulation stage, producing incomplete or contextually imprecise written conclusions.

Nurrahmawati et al. (2021) similarly found in a qualitative study of secondary students that symbolic to verbal translation is a highly error prone representational transition, with students most commonly committing misinterpretation and implementation errors when converting a formally correct algebraic result into a meaningful verbal statement tied to real world context. Dewi et al. (2017) in a study of SMA students examined from a gender perspective, noted that verbal representation defined as the ability to communicate mathematical ideas through written and oral language was the dimension most susceptible to individual differences and the least consistently developed across student groups. Consistent with this body of evidence, S2's verbal performance in the present study stands as a notable counterexample: his spontaneous articulation of the general principle relating a point's computed distance value to the radius classifying positions as inside, on, or outside the circle demonstrates what Mainali (2020) described wherein verbal expression functions not merely as a post-hoc description of procedure but as a medium for conveying relational mathematical structure.

With respect to pictorial representation, S3's performance illustrates a pattern widely documented in the SMA-level mathematics literature: students may demonstrate competent spatial construction specifically, the accurate plotting of a circle’s center and boundary points on a Cartesian coordinate plane. The precise physical drawing of the circle while nonetheless producing visual representations that are communicatively incomplete. S3 constructed a well-organized Cartesian coordinate system, systematically derived four cardinal boundary points from the radius before drawing the circle using a compass for geometric precision and plotted all three test points in positions fully consistent with his symbolic analysis. This principled construction approach is consistent with findings by Sanjaya et al. (2018) who observed that students working on circle material across various learning styles generally succeeded in constructing recognizable geometric figures when given adequate time, but that the completeness and communicative quality of those figures varied substantially. Hertanti et al. (2023) in a classroom action research study involving class XI SMA students on circle equation material, similarly found that students were more confident in performing geometric constructions than in producing fully labeled and self-explanatory diagrams, with labeling accuracy requiring deliberate instructional scaffolding over multiple learning cycles.

The specific shortcoming observed in S3's work the omission of an explicit radius annotation is theoretically significant. According to Mainali (2020), a mathematical diagram constitutes an external representational form that must be interpretable independently of the solver's private understanding; the absence of a labeled radius means that the diagram is not yet a complete semiotic object, since a reader cannot reconstruct the full metric structure of the geometric configuration from the visual alone. This distinction between a diagram that is visually plausible and one that is metrically self-contained represents an important and frequently overlooked dimension of pictorial representational ability. Nurrahmawati et al. (2021) similarly documented this gap in secondary students' graphical representations, noting that students who understood the underlying mathematical concept nonetheless produced incomplete graphs due to insufficient attention to annotational conventions.

Symbolic representation emerged as the most robustly fulfilled domain, though it also revealed meaningful inter-individual differences in strategic approach that carry implications for understanding what symbolic proficiency entails at the SMA level. Both S1 and S4 correctly derived the general equation of the circle, yet through structurally distinct strategies. S1 employed sequential algebraic expansion from the standard form of the circle equation, while S4 used a formula-driven deductive approach by directly computing the general form coefficients using derived substitution formulas. This strategic divergence echoes findings by Sanjaya et al. (2018), who observed that students with theorist and reflective learning styles on circle material were more likely to approach symbolic derivation analytically and systematically, whereas pragmatist and activist learners tended toward more direct formula-based substitution both yielding correct results but reflecting qualitatively different cognitive engagements with the symbolic structure. Mainali (2020) emphasized that symbolic fluency is best understood not as a single procedure but as a flexible repertoire of equivalent strategies, and the present study confirms that this multi-strategy landscape is observable even within a small sample of SMA students on circle equation problems.

S2's systematic application of the substitution expression for all three test points, paired with verbal articulation of the relational logic comparing the computed value to r^2, exemplifies what Nurrahmawati et al. (2021) identified as a marker of high-quality symbolic representation: coherence between formal symbolic operation and conceptual verbal justification. A notable exception was S1's occasional use of a colon (:) in place of the equality sign (=) a convention inconsistency that points to incomplete internalization of mathematical notational standards. Dewi et al. (2017) noted that such micro level inconsistencies in symbolic notation are more common among SMA students than is often assumed, and tend to persist even after students have mastered the broader procedural steps of a given problem, suggesting that notational precision merits explicit instructional attention independent of procedural correctness.

Taken together, the findings of this study carry important theoretical and practical implications for SMA level mathematics education, particularly for the teaching and learning of circle equations as a topic that spans all three representational modalities in an integrated way. Theoretically, this study contributes to the body of evidence reviewed systematically by Lutfi & Juandi (2023) establishing that representational ability among Indonesian high school students is consistently characterized by uneven inter-modal profiles, with symbolic representation most developed, verbal representation most vulnerable, and pictorial representation susceptible to incomplete annotation. This hierarchy has been documented specifically for circle material by Hertanti et al. (2023) and Sanjaya et al. (2018), and the present study extends and deepens this evidence through close qualitative analysis of individual students' representational processes on contextual circle equation problems.

The finding that verbal representation constituted the most differentiated dimension across subjects with S1 exhibiting imprecision, S2 demonstrating conceptual depth, and S3 performing at an adequate level corroborates the conclusion of Dewi et al. (2017) that verbal representational ability is the dimension whose performance varies most markedly in accordance with individual ability and motivational level among SMA students. That is, unlike other representational dimensions, which tend to remain relatively stable across students of differing aptitude, verbal representation exhibits a comparatively wide performance gap, rendering it the dimension most responsive to learner-level variation. Consequently, this dimension warrants deliberate and targeted pedagogical attention rather than being left to develop incidentally as a by-product of instruction focused on procedural or visual representation. From a practical standpoint, these findings suggest that mathematics instruction on circle equations at the SMA level should explicitly and systematically scaffold three components: (a) the verbalization of geometric meaning within contextual language, requiring students to move beyond stating computational results toward articulating their geometric and real-world significance; (b) adherence to the completeness conventions of mathematical diagrams, including the explicit annotation of all relevant metric properties such as the radius; and (c) the consistent and accurate use of mathematical notation, treated as a discrete learning objective rather than an incidental outcome of computational proficiency.

Hertanti et al. (2023) demonstrated in a classroom action research setting that guided discovery learning applied over two cycles on circle equation material produced significant gains in students' representational ability specifically in the direction of more complete and precise representations suggesting that instructional intervention of this kind is a viable pathway for addressing the gaps identified in the present study. Future research should address the limitations of this study's qualitative and small-scale case design by employing larger samples drawn from diverse SMA contexts across Indonesia. Comparative studies examining whether the verbal, pictorial, symbolic representational hierarchy identified here generalizes across other SMA topics involving coordinate geometry such as conics, ellipses, or lines and circles would further contribute to the evidence base needed to inform curriculum and assessment design.

CONCLUSION

This study set out to describe the mathematical representational abilities of SMA students in solving a contextual problem involving circle equations, with particular attention to three representational modalities: verbal, pictorial, and symbolic. Symbolic representation was the most completely fulfilled domain, as evidenced by students' accurate derivation of the general circle equation and correct application of the distance-based substitution formula to determine point positions though with inter-individual variation in strategic approach and notational precision. Pictorial representation was adequately fulfilled in terms of spatial construction, with one student successfully deriving cardinal boundary points from the radius and plotting all test points in geometrically consistent positions; however, the omission of an explicit radius annotation rendered the diagram communicatively incomplete as a self-contained mathematical object. Verbal representation proved to be the most uneven domain: while all students could identify given information from the contextual problem, only the highest-performing student succeeded in articulating the relational geometric principle underlying point classification in precise and contextually accurate language, whereas another student committed a meaningful verbal inaccuracy by conflating the geometric referent with a colloquial term, reflecting a disconnect between procedural correctness and semantic understanding. Theoretically, the study affirms that mathematical representation is a multidimensional competency in which symbolic, pictorial, and verbal modalities function as distinct cognitive domains requiring independent instructional attention, consistent with established frameworks in mathematics education research. The study further substantiates that the symbolic to verbal translation pathway constitutes a particularly vulnerable representational transition for SMA students, even when symbolic computation is correctly performed, and that diagram completeness specifically the annotational sufficiency of geometric figures represents a dimension of pictorial representational ability that is systematically underdeveloped relative to spatial construction ability. Practically, these findings suggest that instruction on circle equations at the SMA level should move beyond procedural symbolic competency to explicitly scaffold students' capacity to verbalize geometric meaning in contextual language and to produce metrically complete and communicatively self-sufficient diagrams. Future research should extend this work through larger-scale studies involving diverse SMA contexts, as well as through experimental or longitudinal designs that examine how targeted multi-modal instructional interventions particularly those emphasizing verbal articulation and annotational completeness can develop more balanced representational competency in SMA mathematics learners.

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